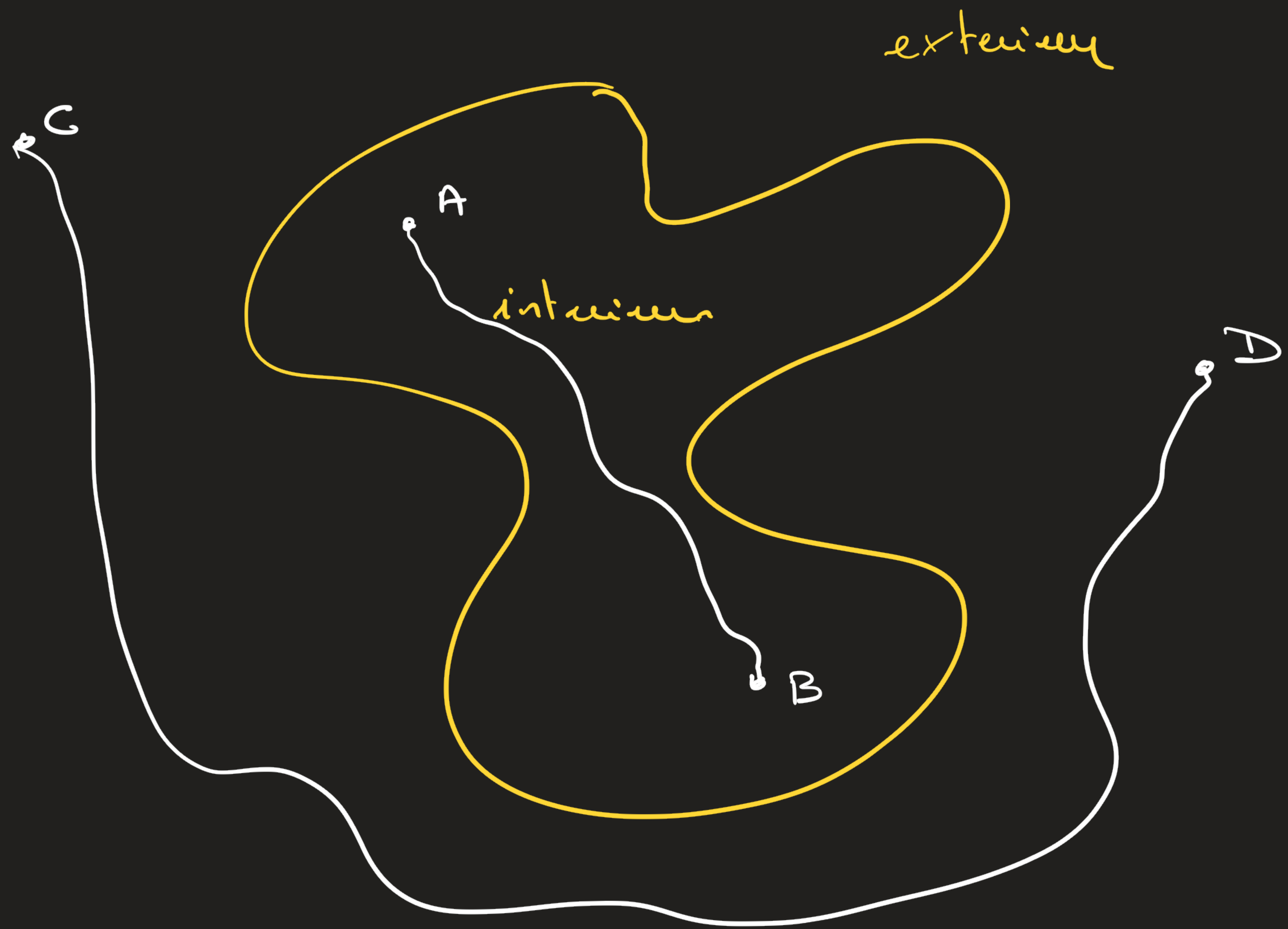
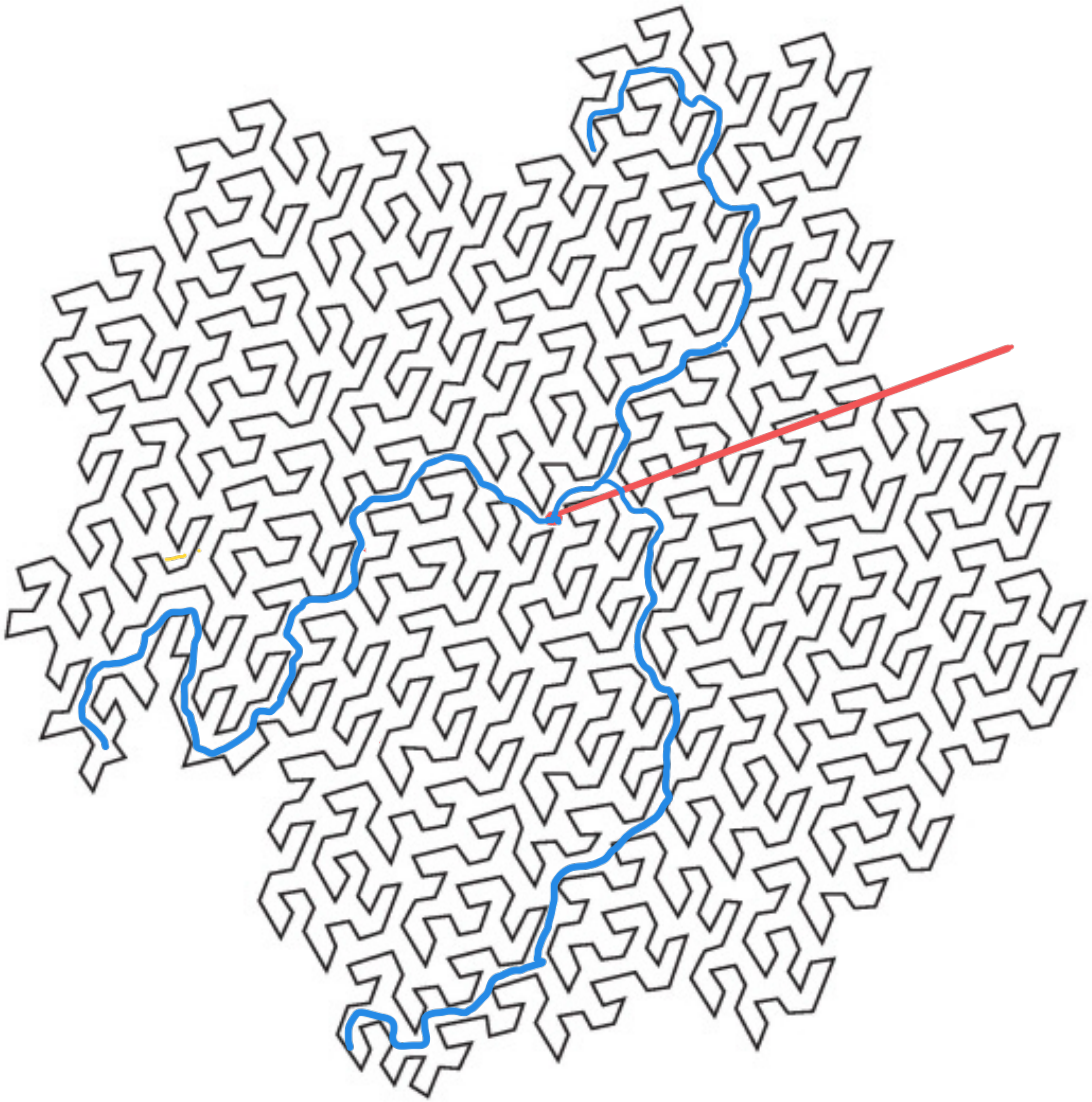
Théorème de Jordan.

Théorème (de Jordan)

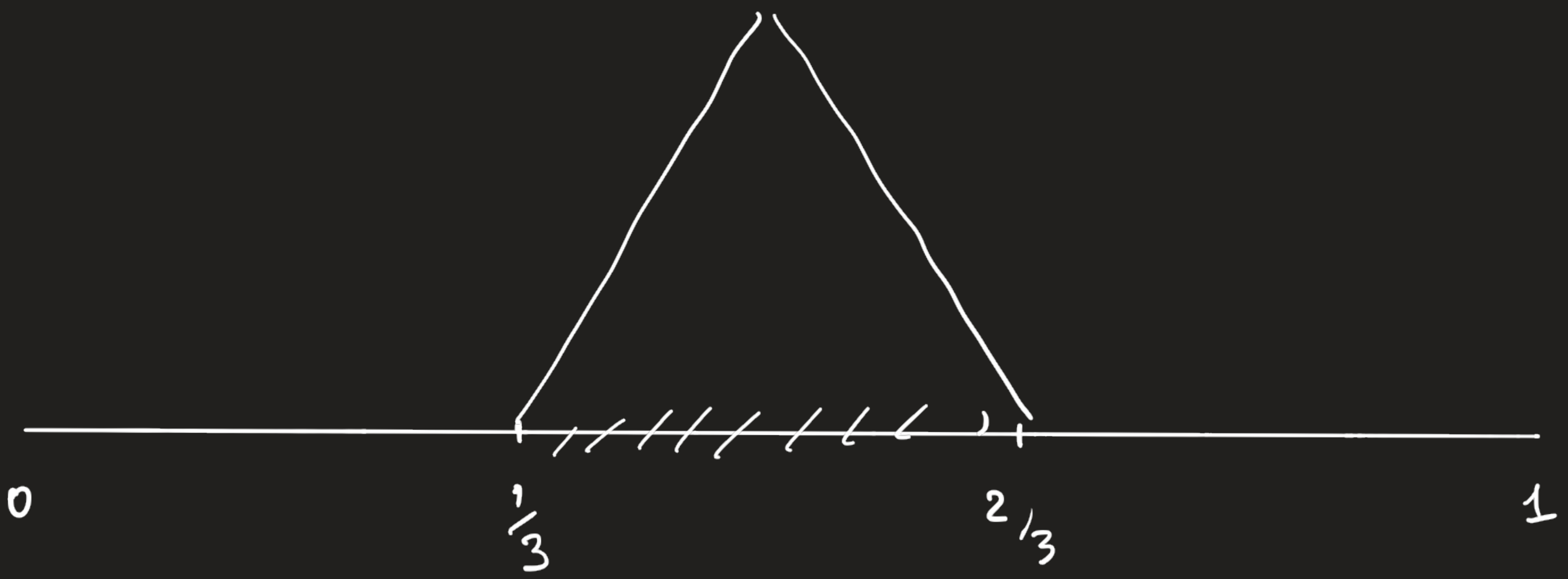
Une courbe de Jordan sépare le plan en <u>deux</u> composantes connexes disjointes.


extérieur
C
A
intérieur
D
B



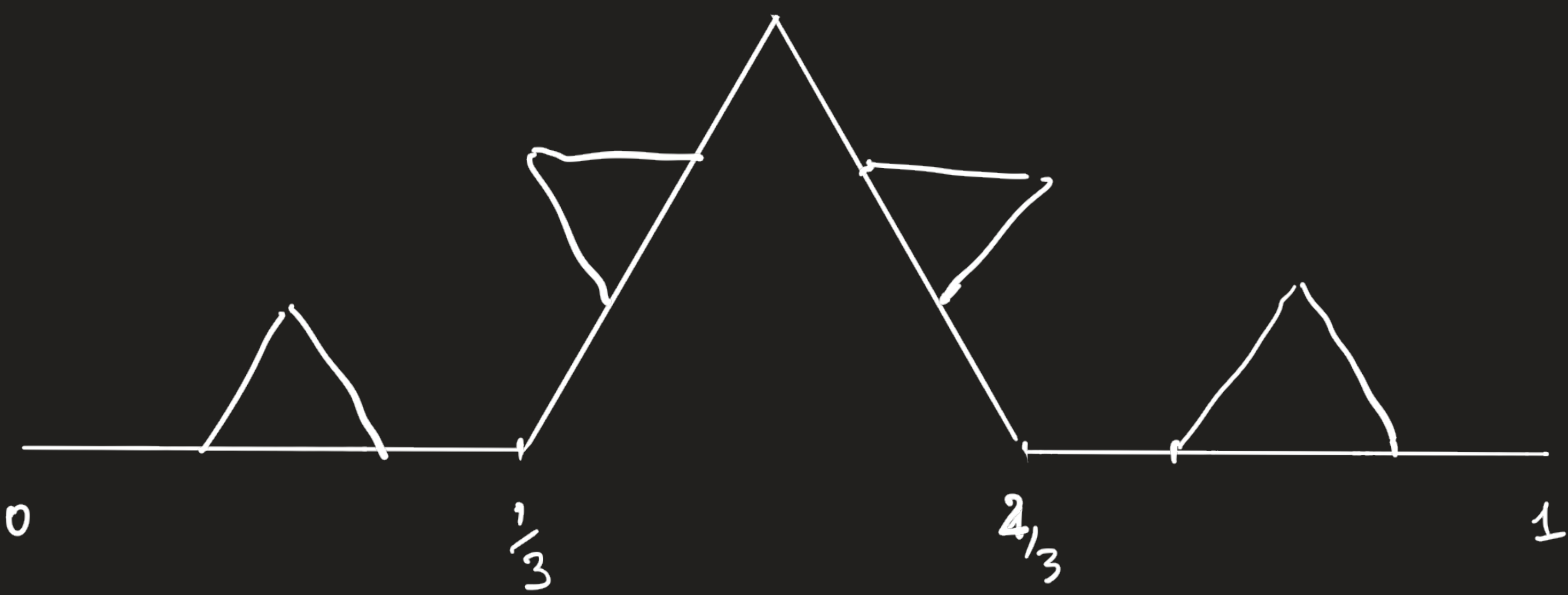
Courbe de Koch:

$L = 1$



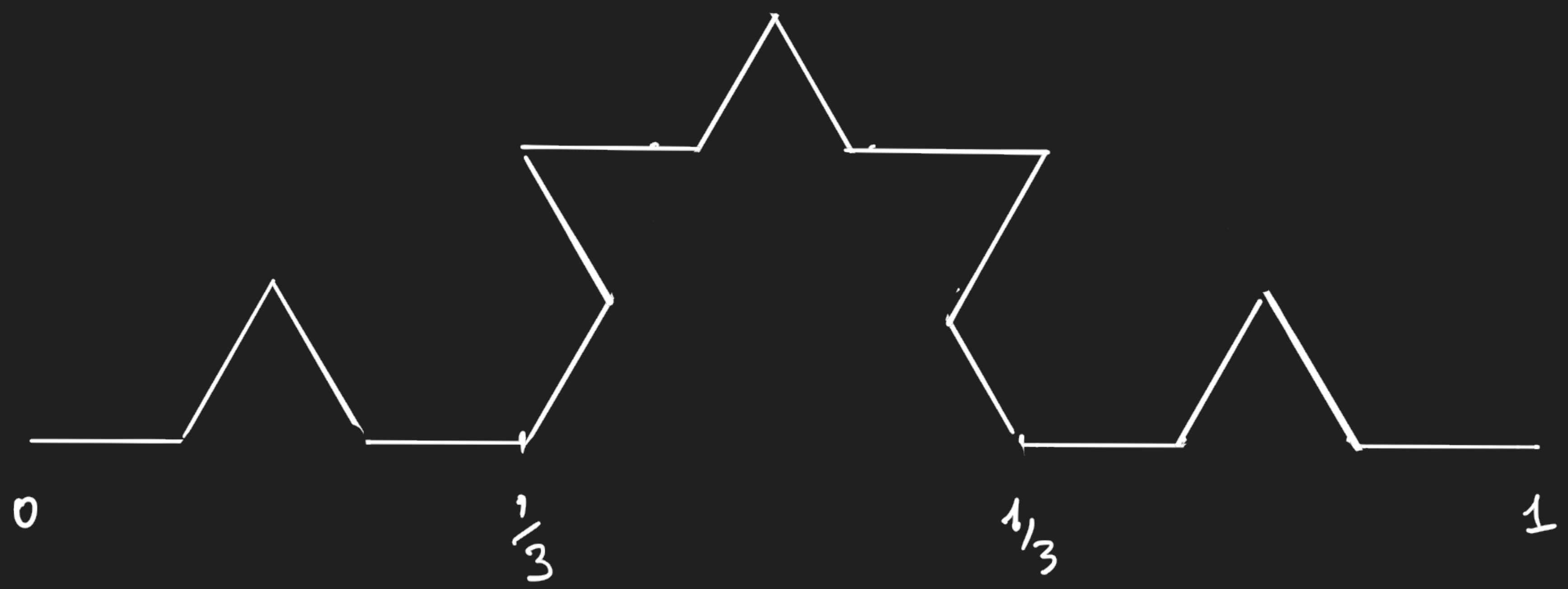
Courbe de Koch :

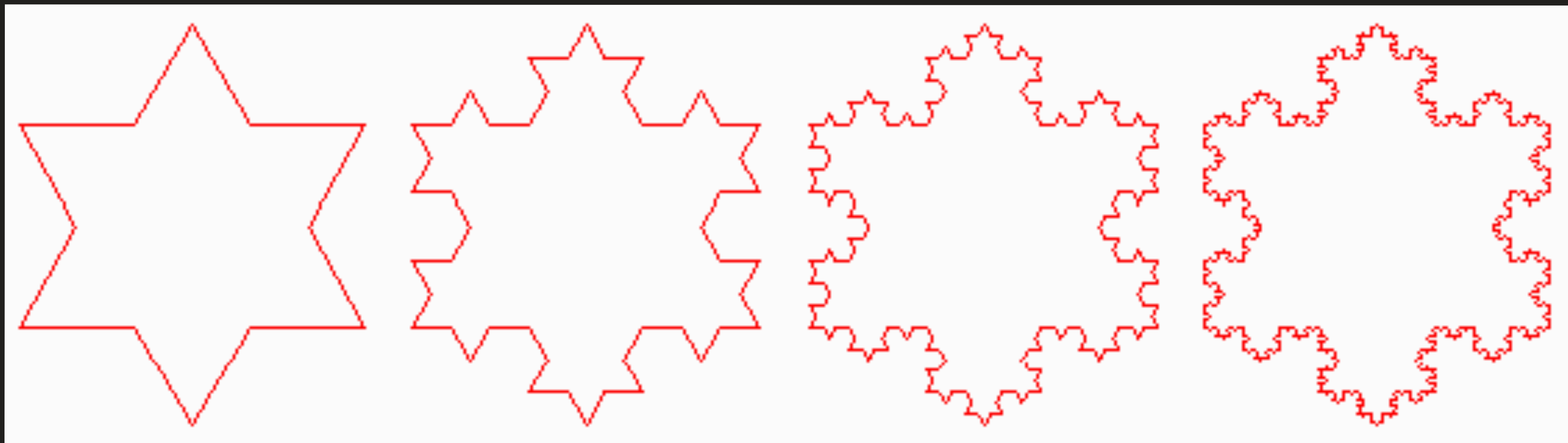
$$L = \frac{4}{3}$$

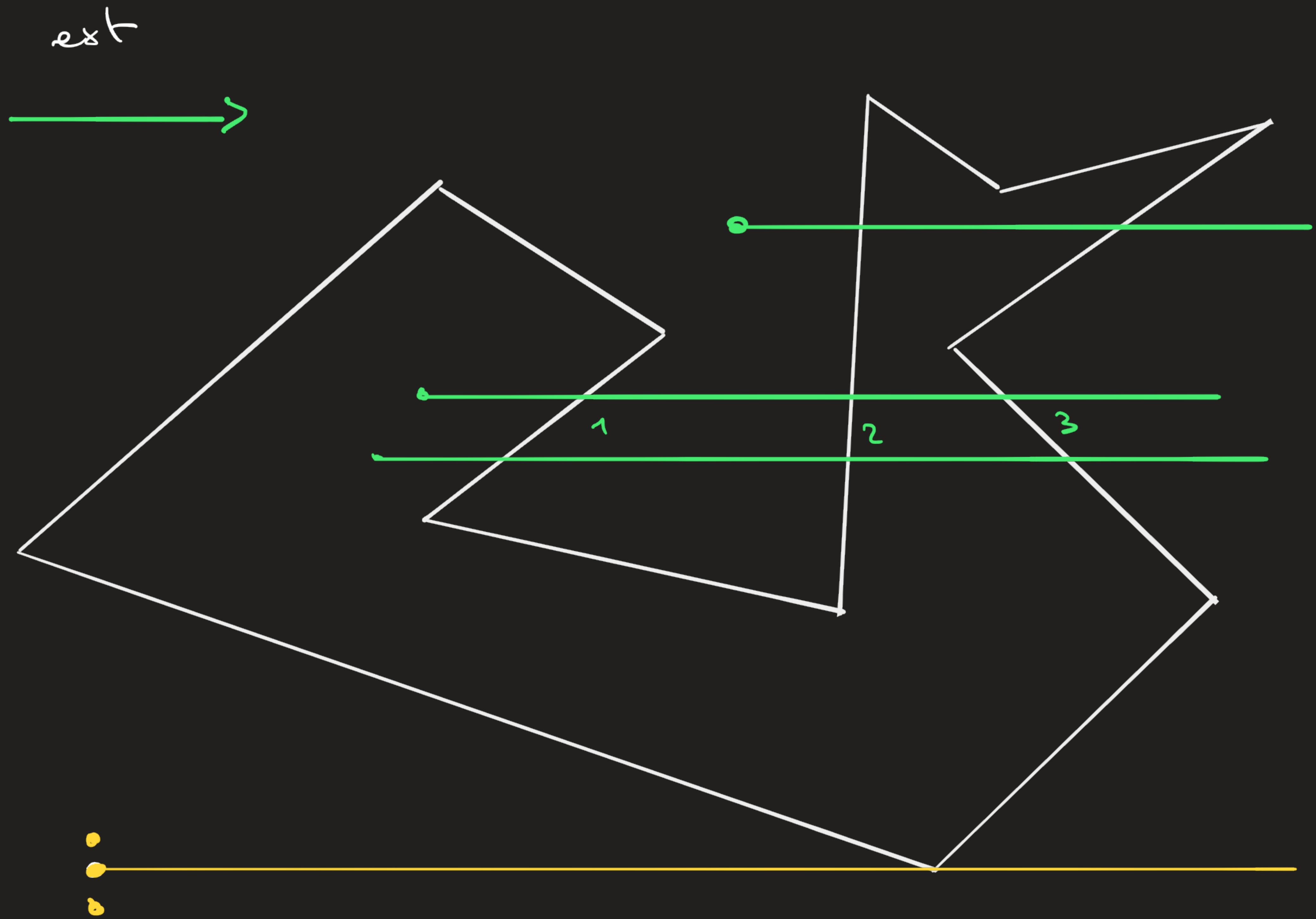


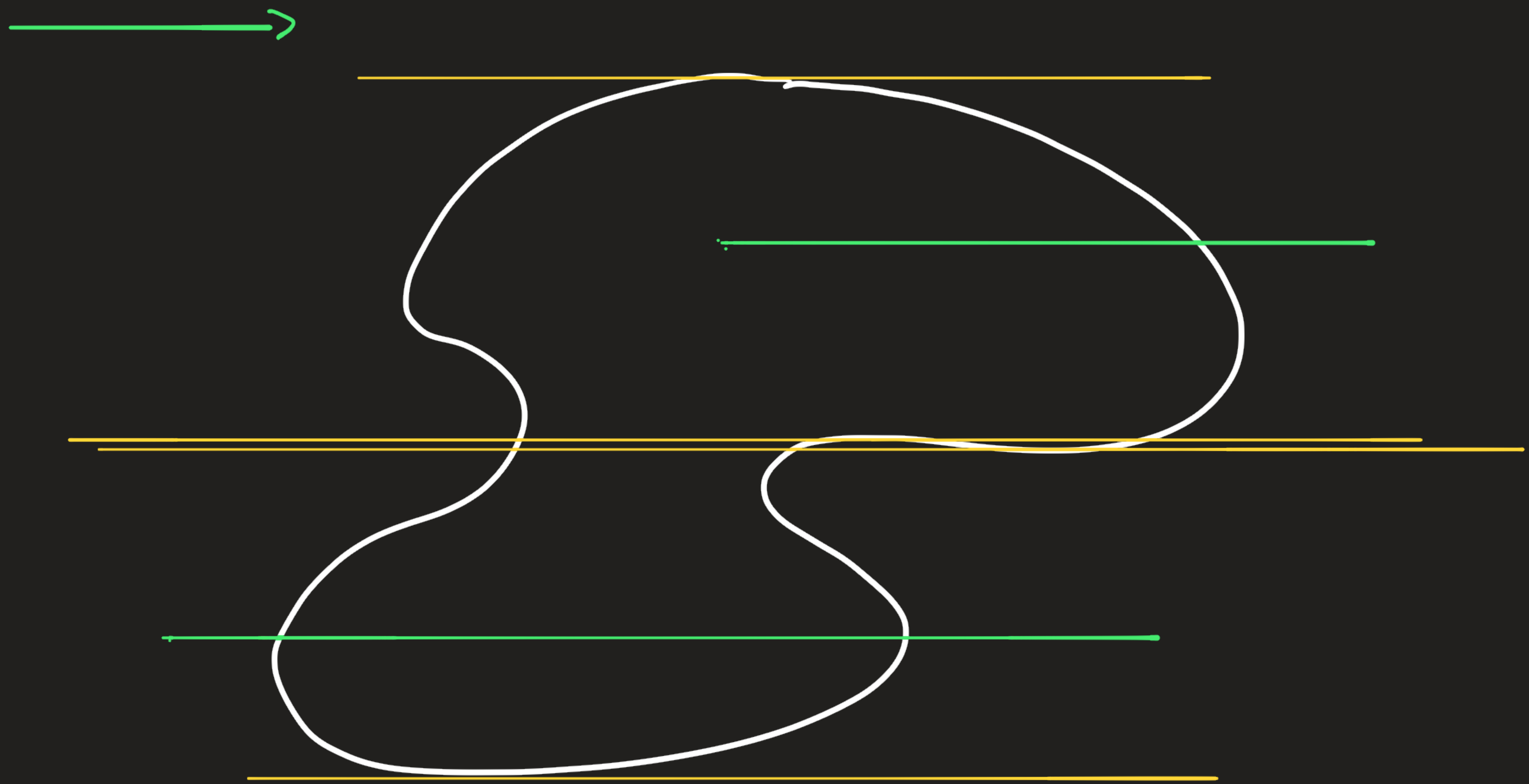
Courbe de Koch :

$$L = \left(\frac{4}{3}\right)^2$$






ext
1
2
3



Régularité des fonctions.

- Continues
- Höldériennes $\quad x \longmapsto \sqrt{|x|}$
- Lipschitziennes $\quad x \longmapsto |x|$
- Dérivables $\quad x \longmapsto x^2$
- Continument dérivables
- n- fois dérivables
- ∞ - fois dérivables
- analytiques.

Régularité des fonctions

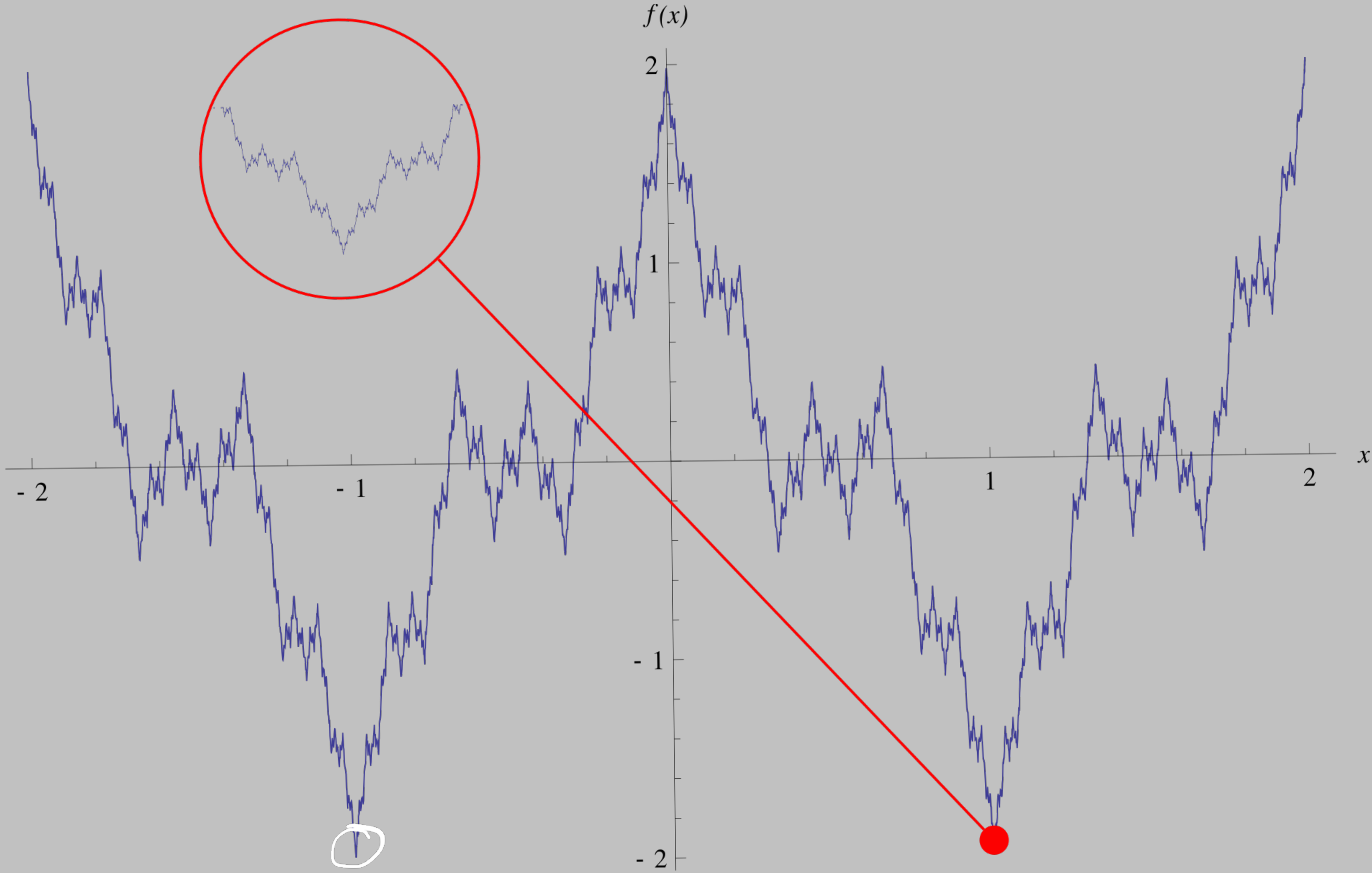
Fonction de Weierstrass.

$$f(x) = \sum_{n=0}^{+\infty} \frac{1}{2^n} \cos(3^n \pi x)$$

$x \in \mathbb{R}$

continue partout

nulle part dérivable


f(x)
x
- 2
- 1
1
2
2
1
- 1
- 2

la Conjecture de Toeplitz.

Histoire

1911 Toeplitz

1916 Arnold Emch. (Convexes)

1929 Lev Schnirelman (courbes $\mathcal{C}^2$)

⋮

2017 Terence Tao. (lipschitziennes)

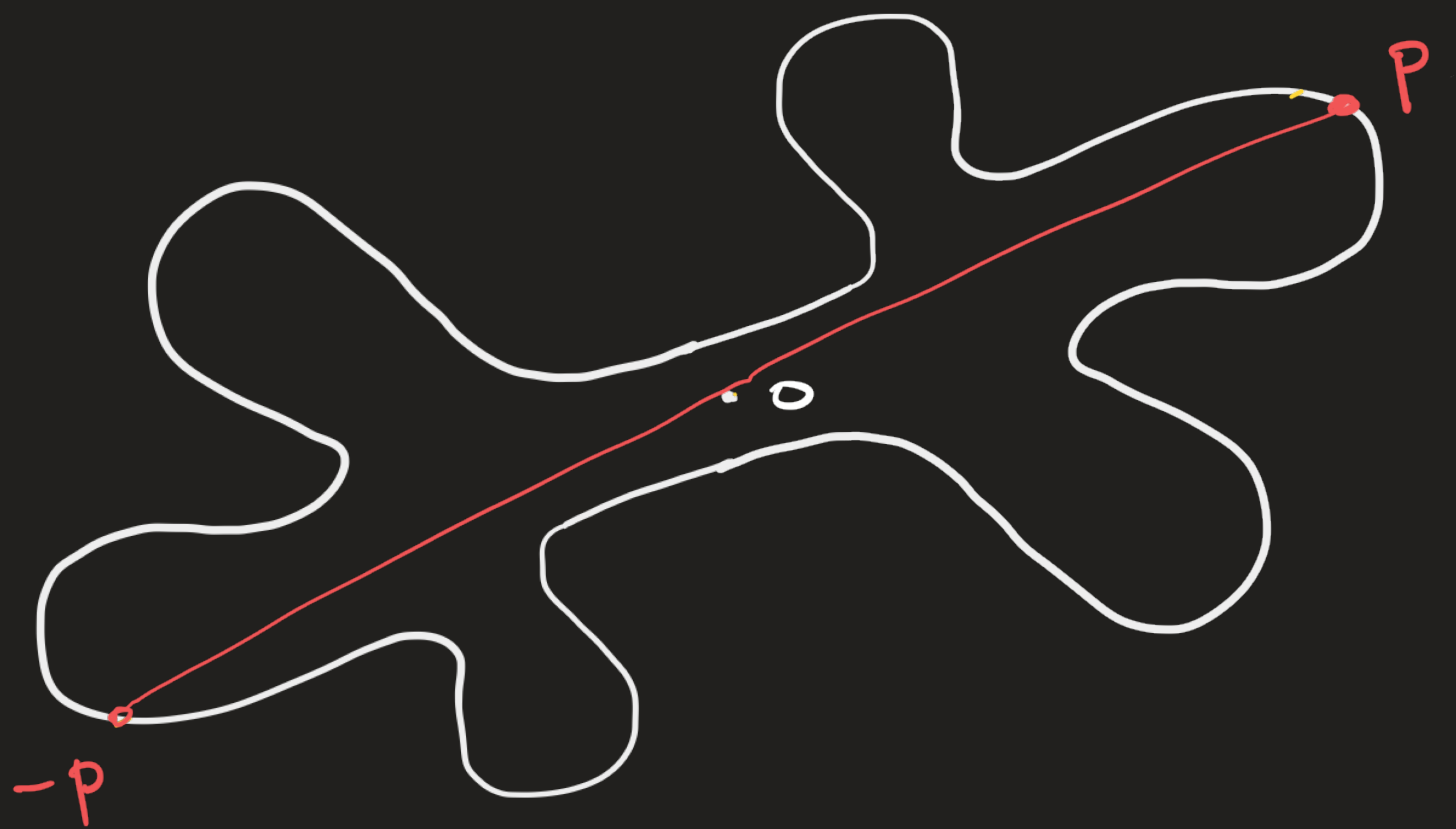
$$|f(x) - f(y)| \leq c\,|x-y|$$

Stratégie pour le cas général

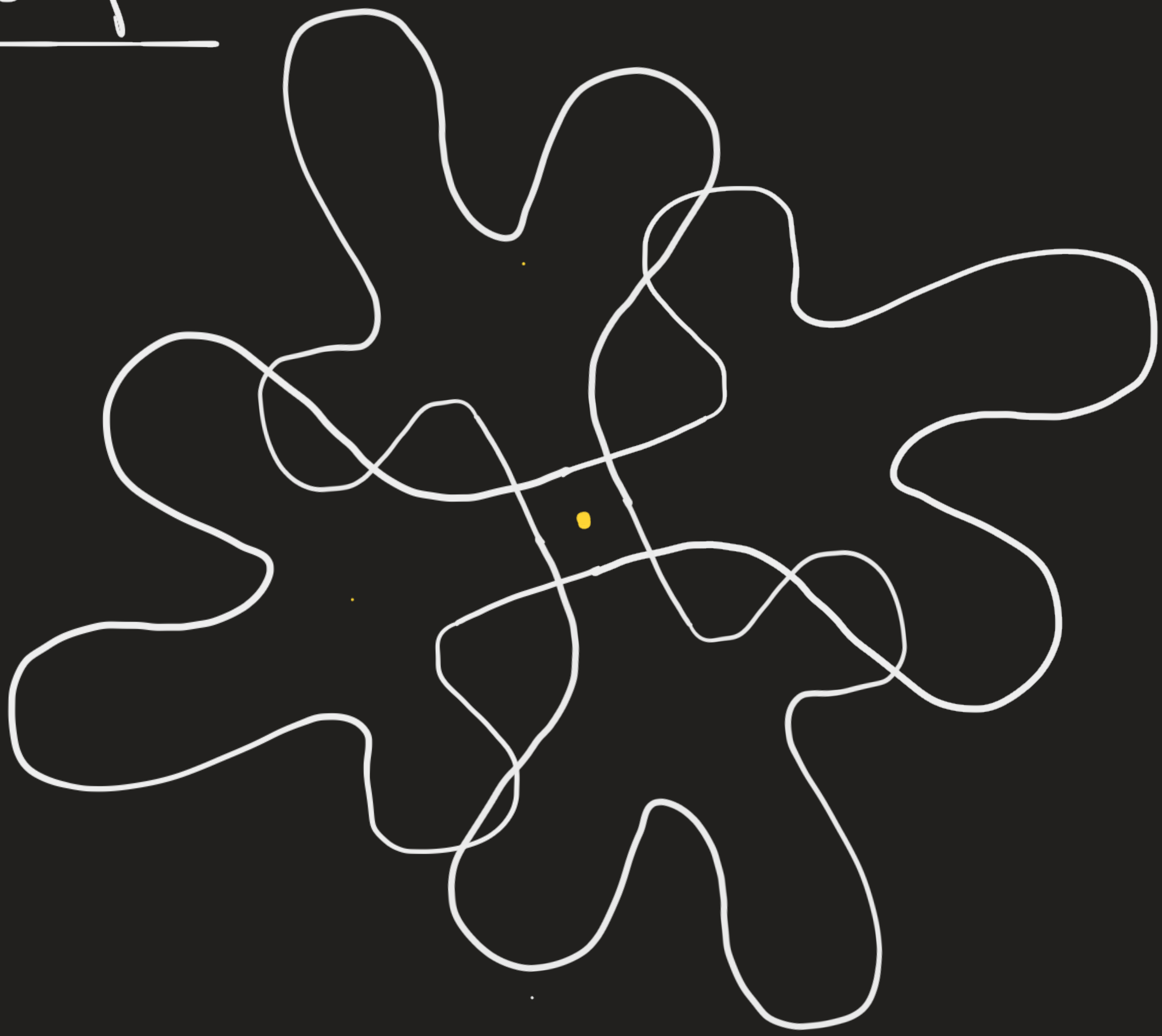
① Approcher Γ par une suite de courbes plus régulières

② Passer à la limite.

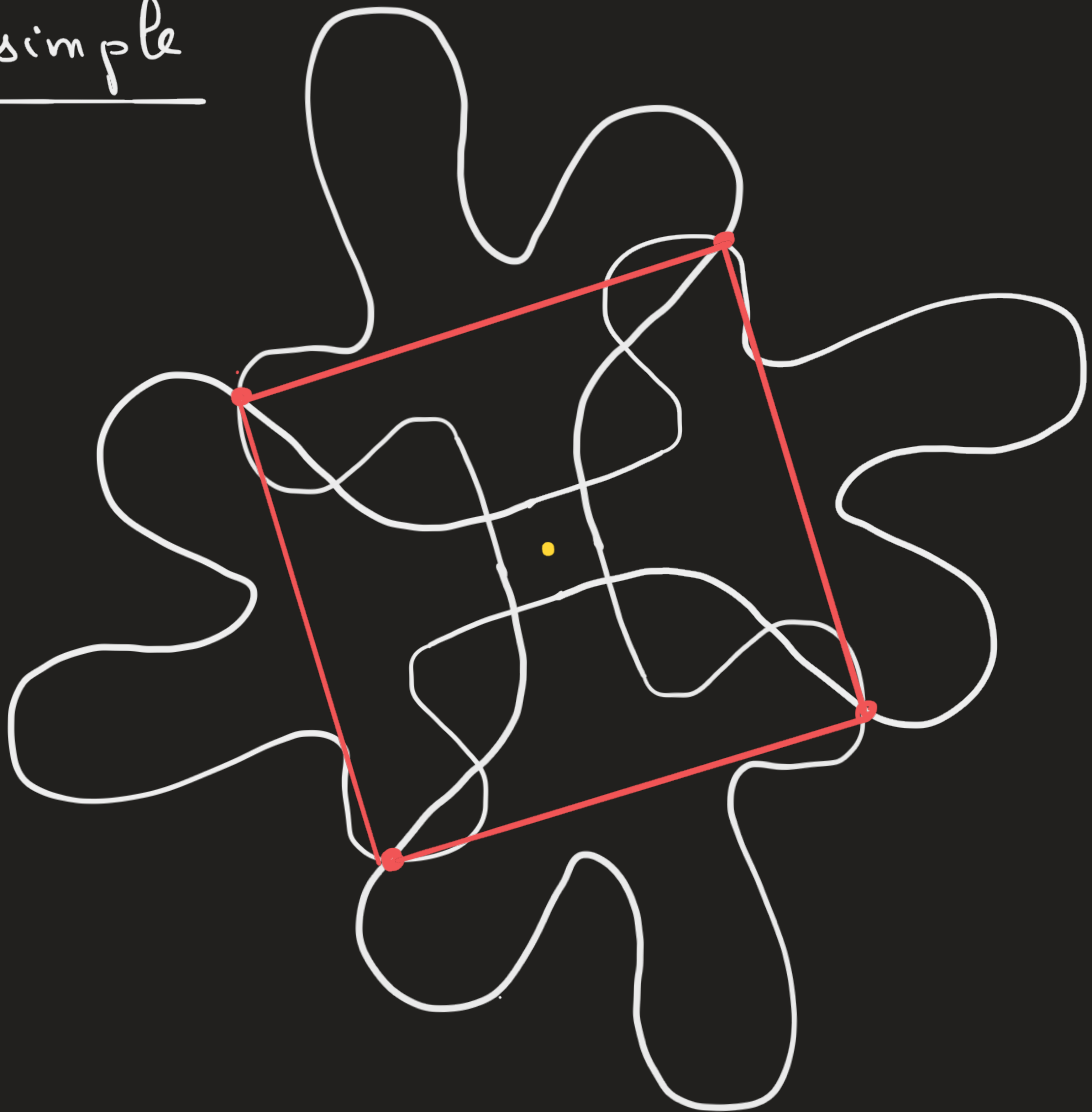
Un cas simple



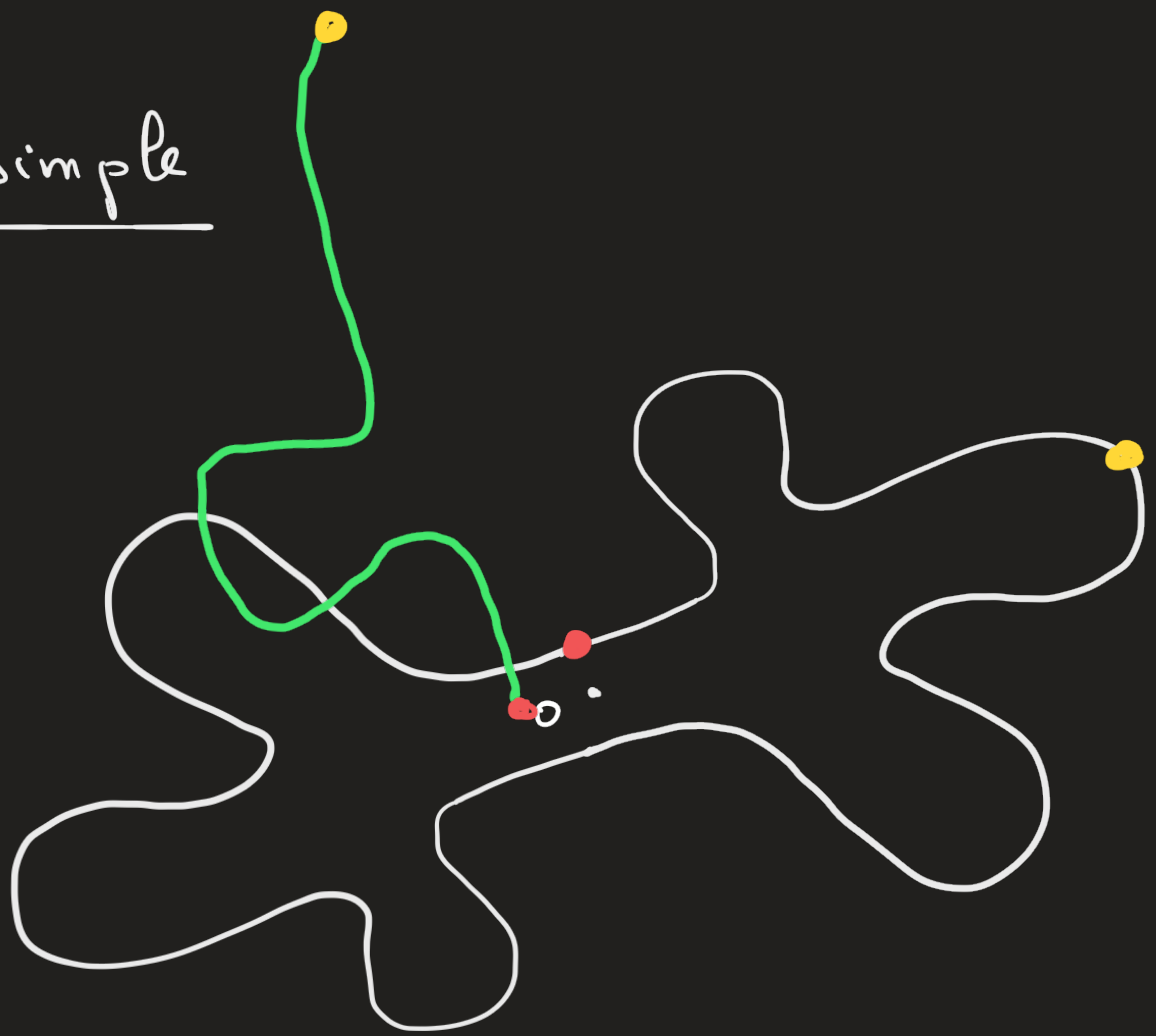
Un cas simple



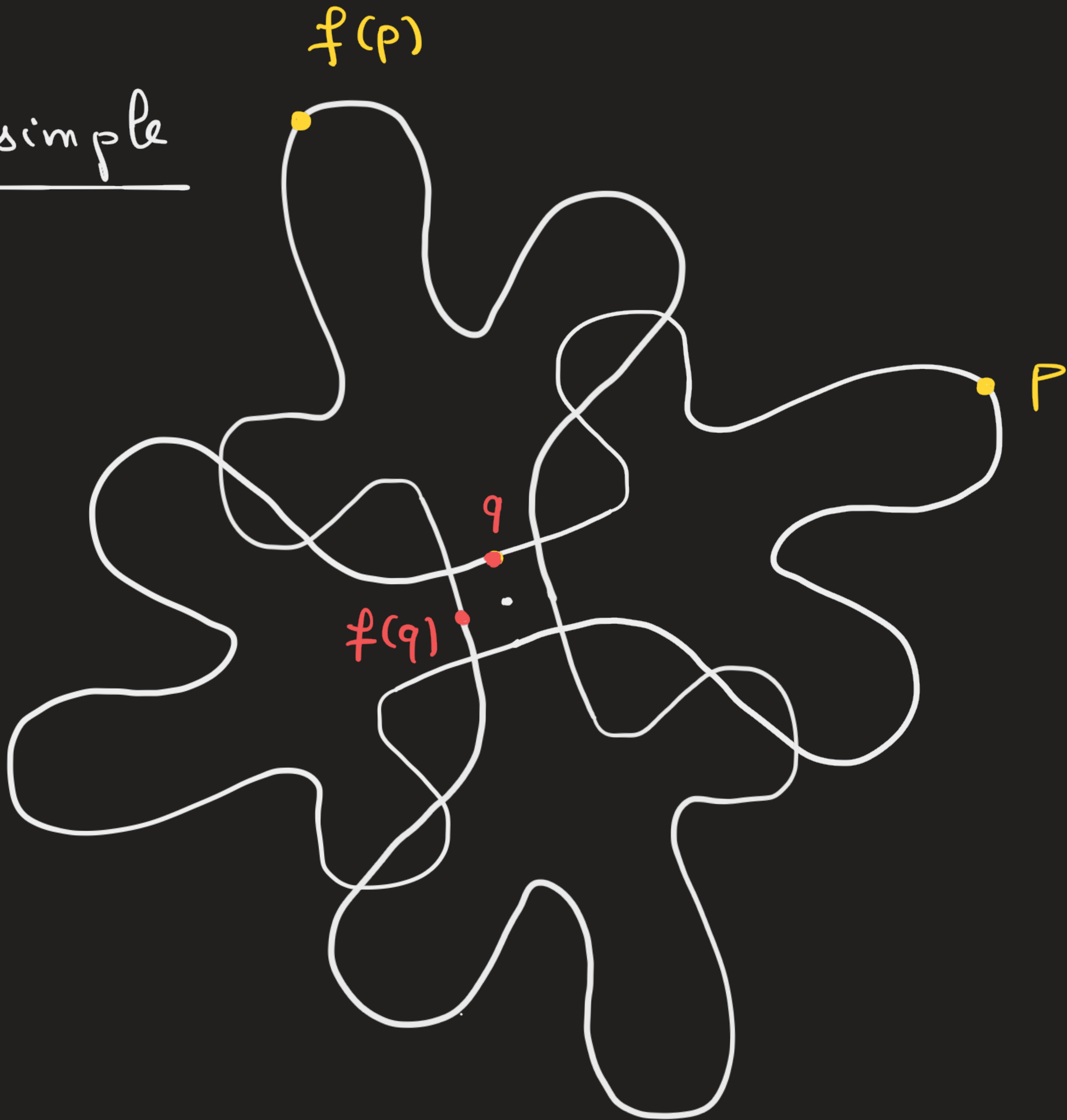
Un cas simple



Un cas simple



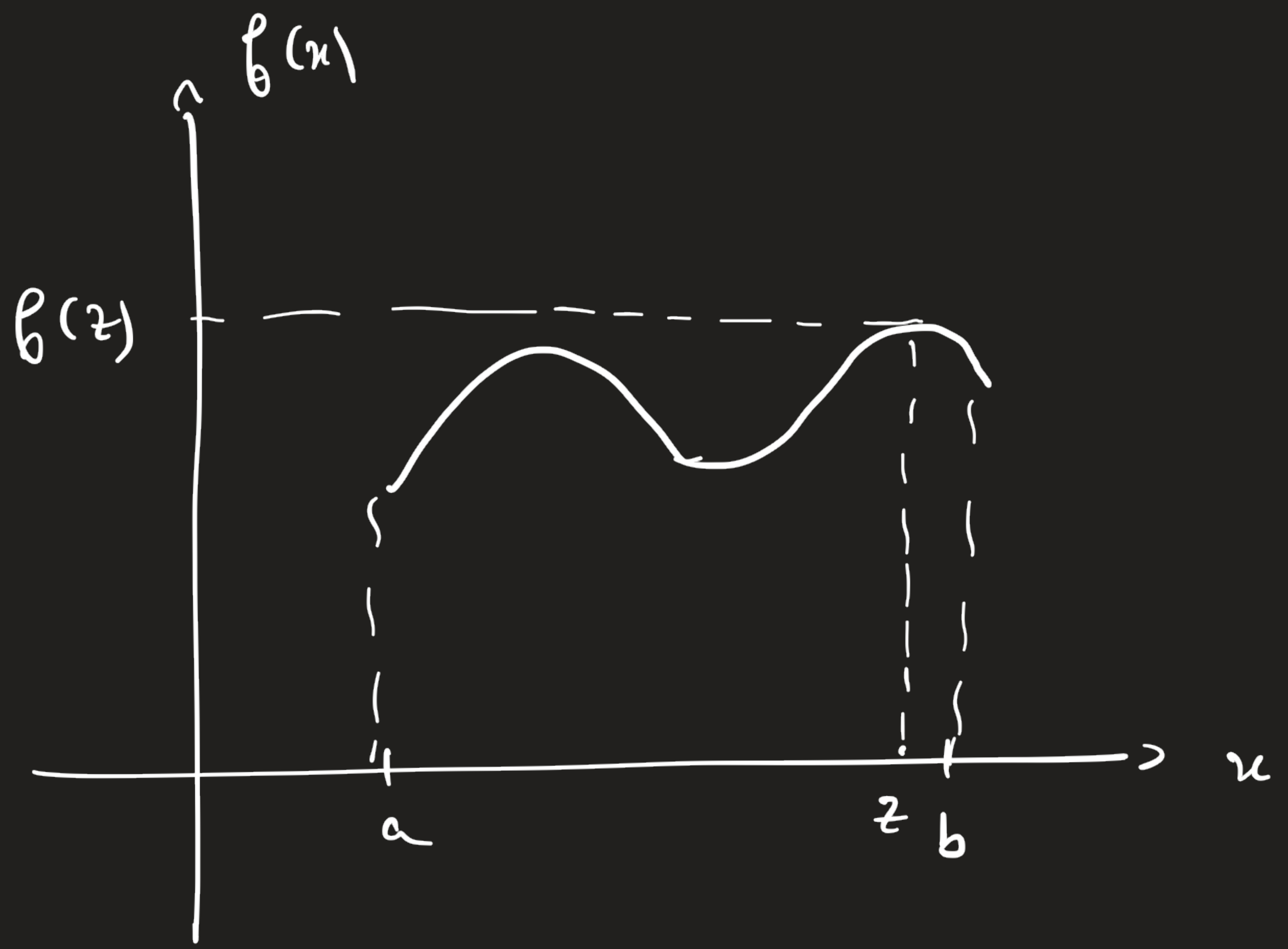
<u>Un cas simple</u>

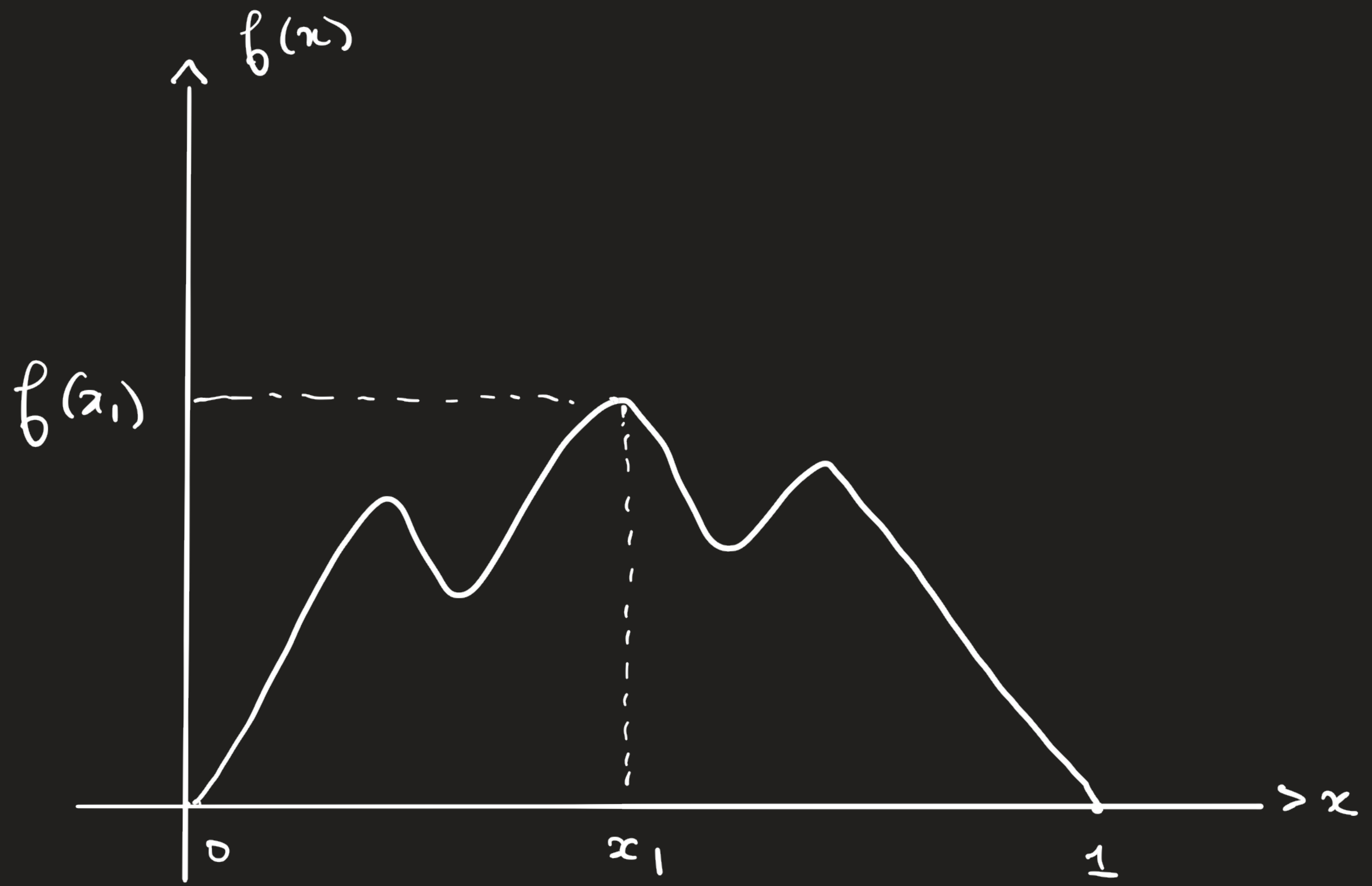


Théorème de la borne atteinte

Théorème Soit $f : [a,b] \longrightarrow \mathbb{R}$ continue alors f est bornée sur $[a,b]$ et il existe $z \in [a,b]$ tq $f(x) \leq f(z)$ pour tout $x \in [a,b]$

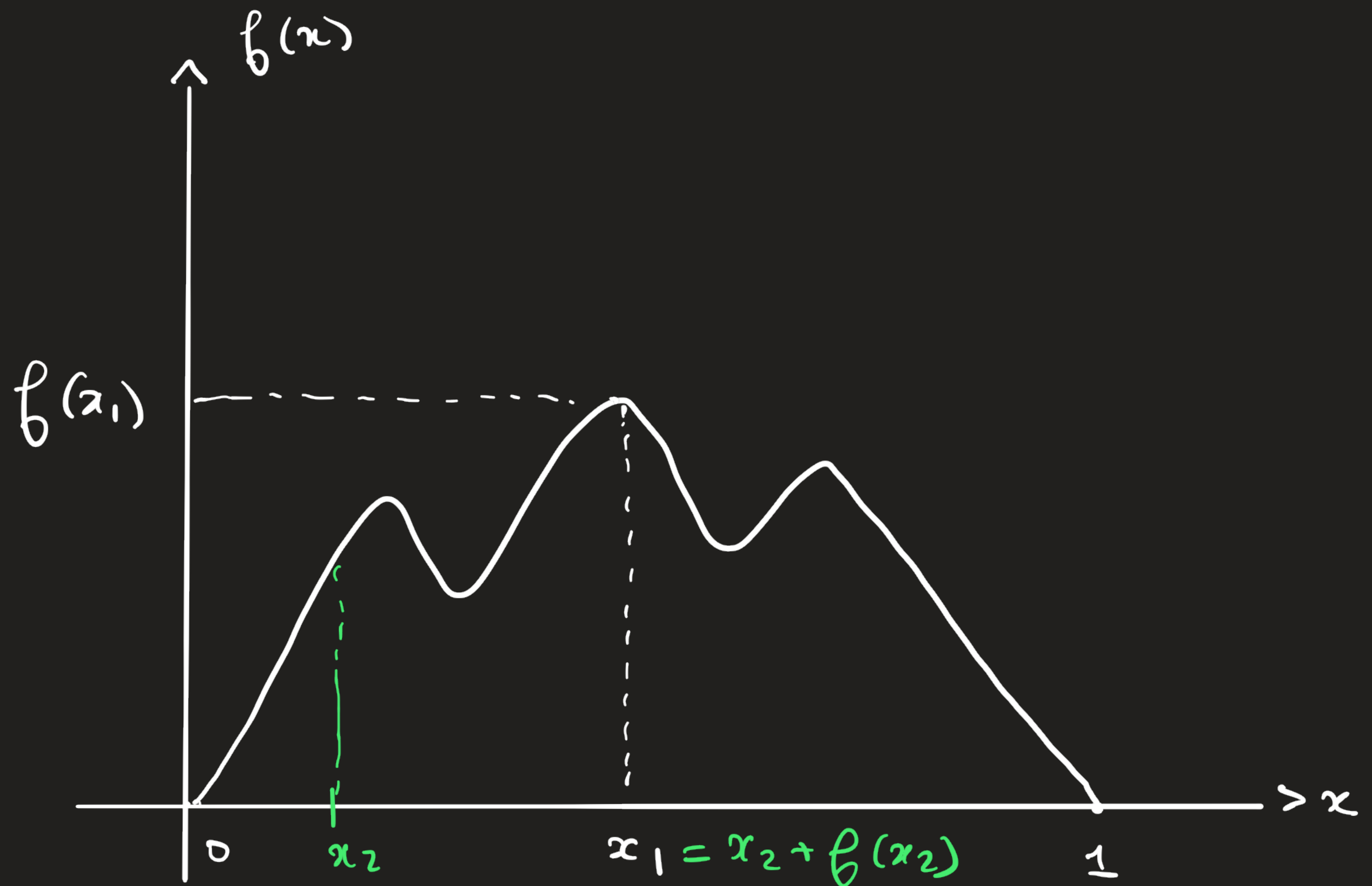
$$f(z) = \max_{[a,b]} f.$$


f(x)
f(z)
a
z
b
x



Choisissons $x_1 \in]0,1[$ tel que

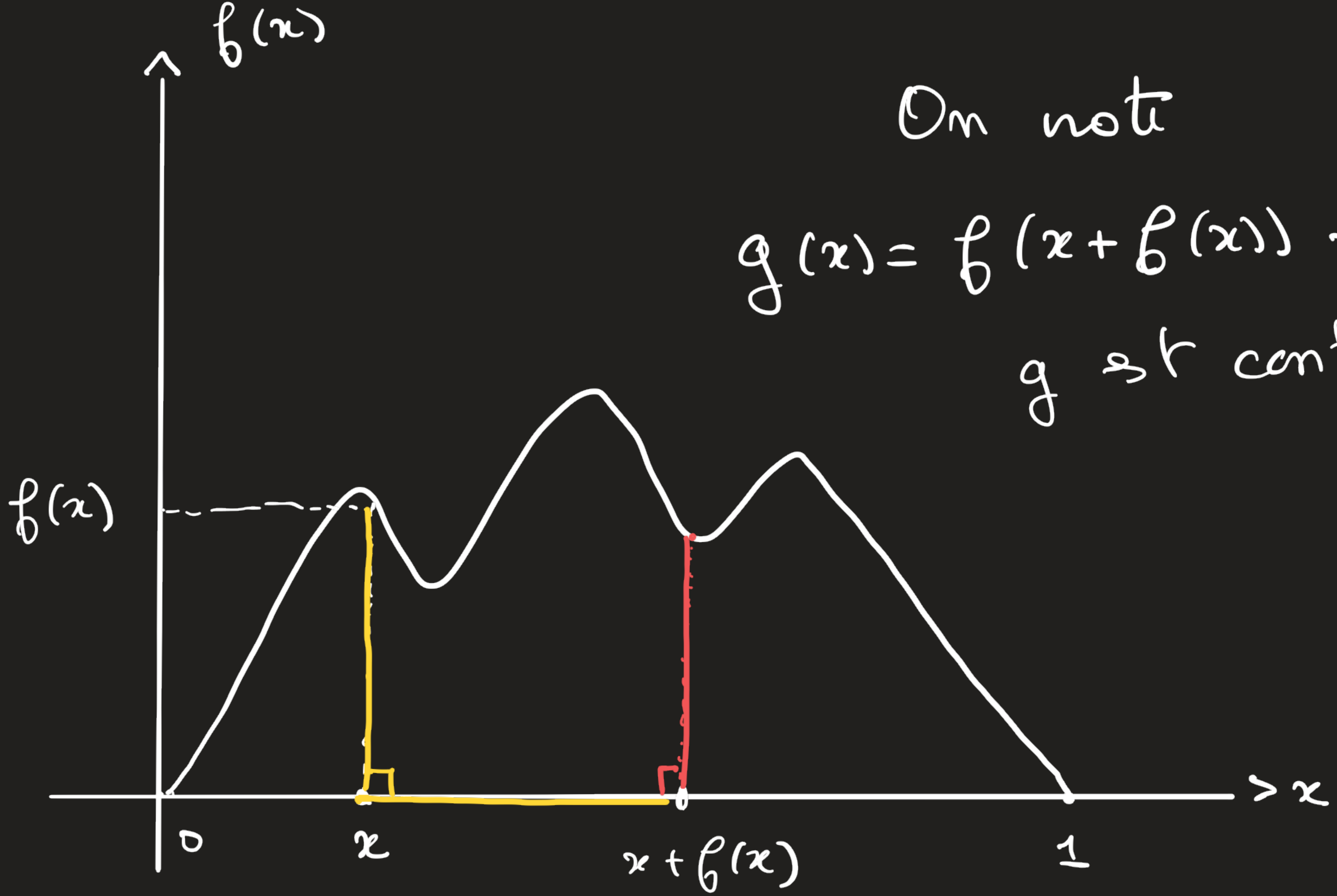
$$f(x_1) = \max_{[0,1]} f$$



On note $h(x) = f(x) + x$

h est continue et $h(0) = 0$ $h(1) = 1$

donc il existe $x_2 \in]0, 1[$ $h(x_2) = x_1$



On noté

$$g(x) = f(x + f(x)) - f(x)$$

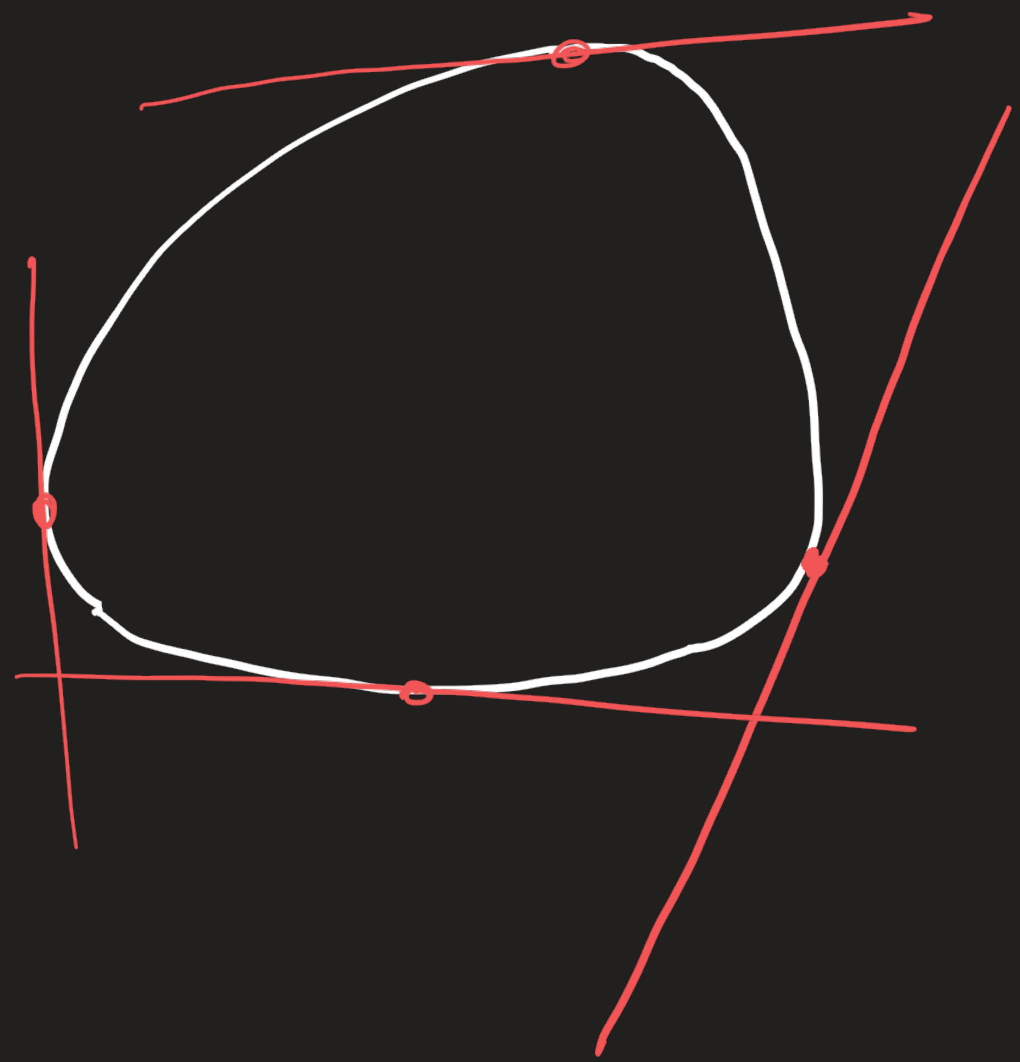
g est continue

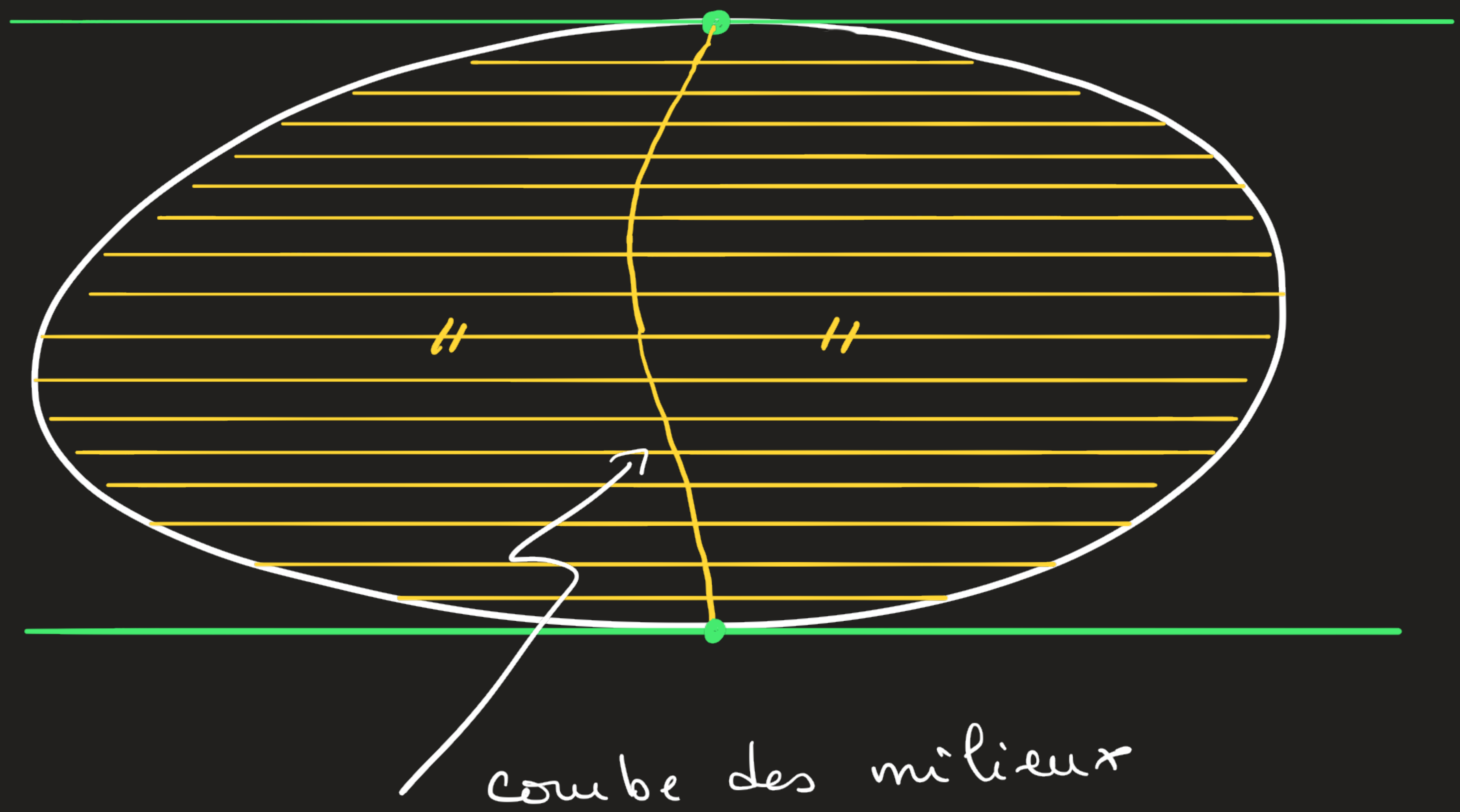
$$g(x_1) = f(x_1 + f(x_1)) - f(x_1) \leq 0$$

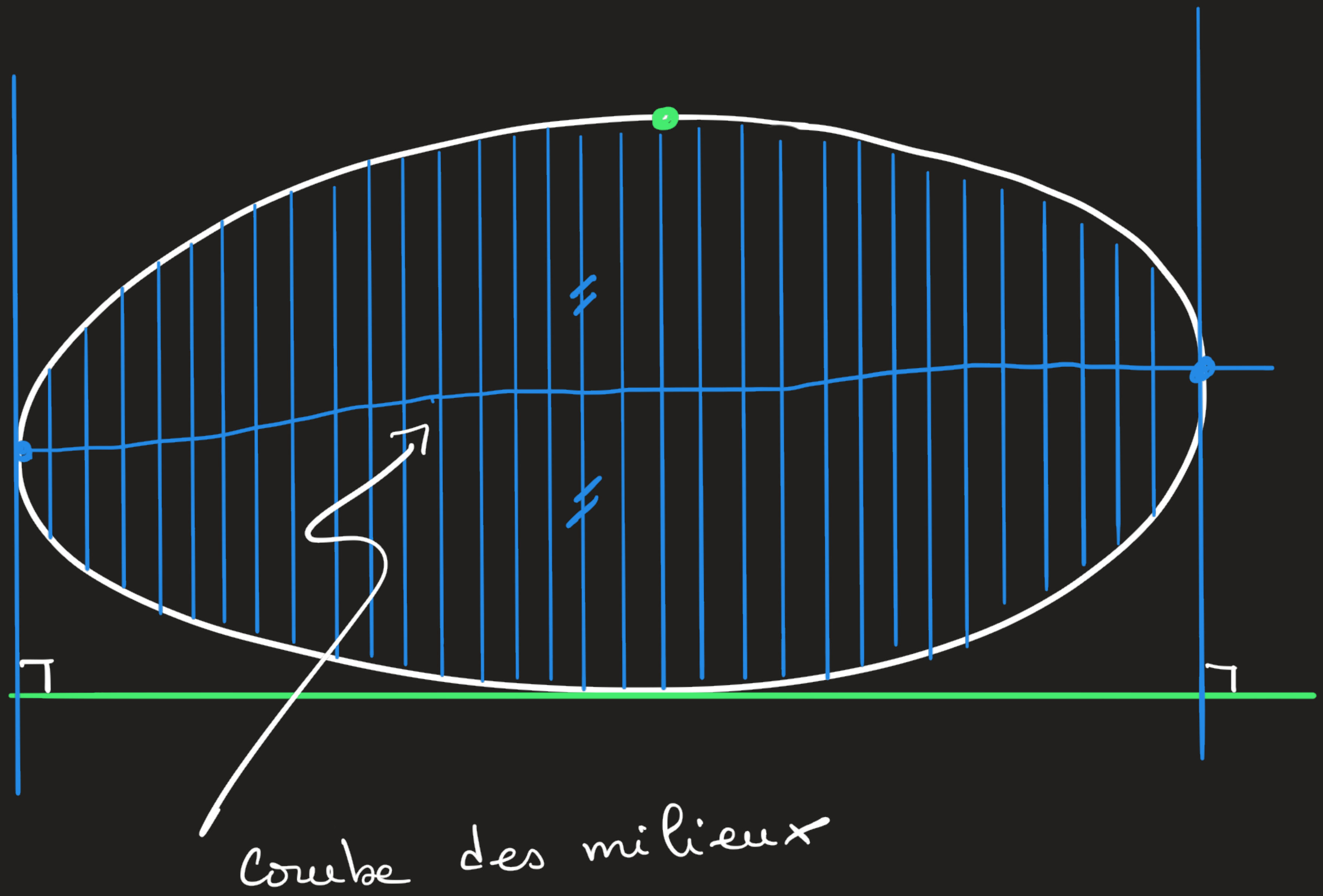
$$g(x_2) = f(x_1) - f(x_2) \geq 0$$

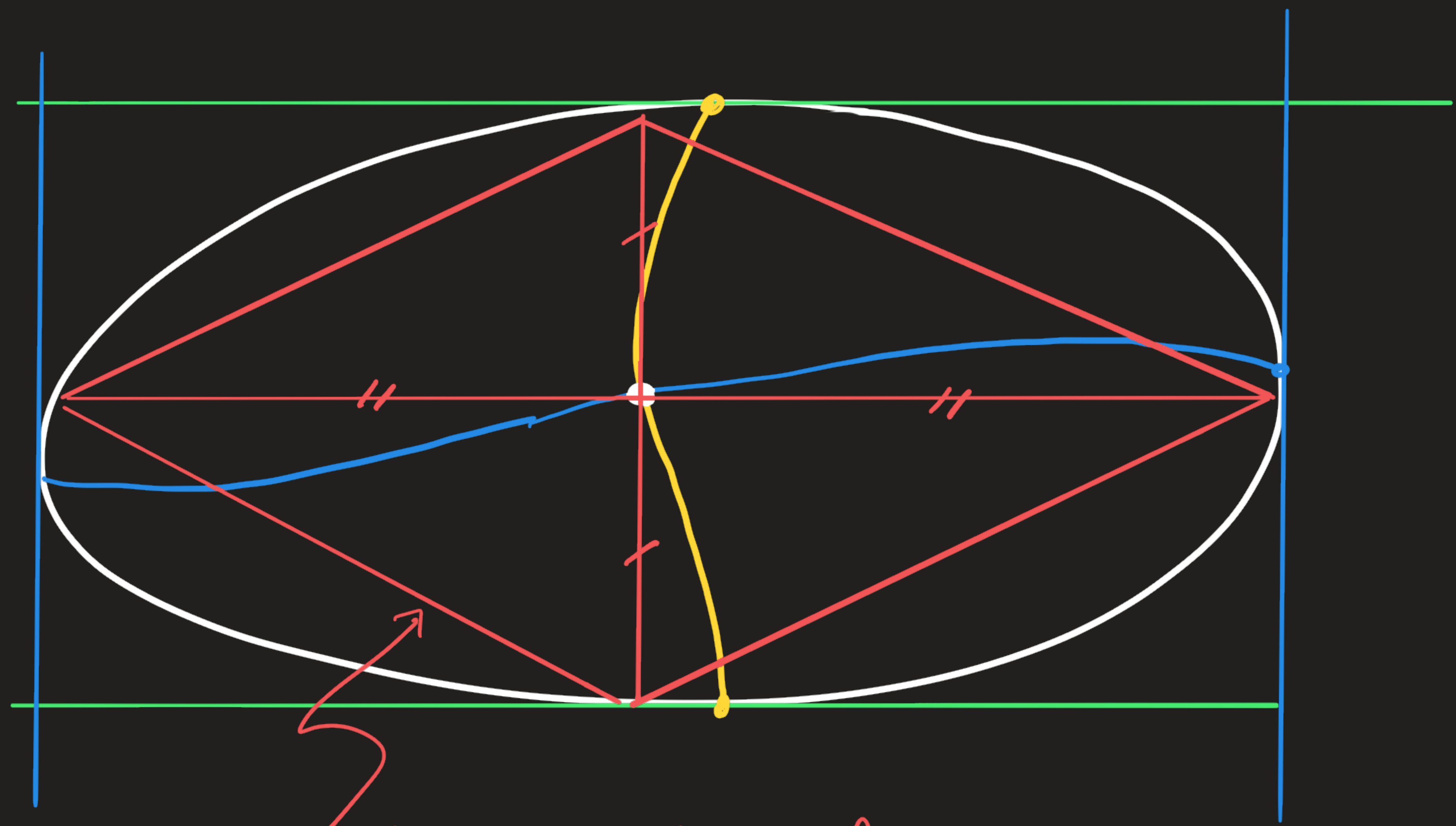
Donc (TVI) il existe $x \in]x_2, x_1[$ tq $h(x) = 0$.

Le cas d'une courbe strictement convexe

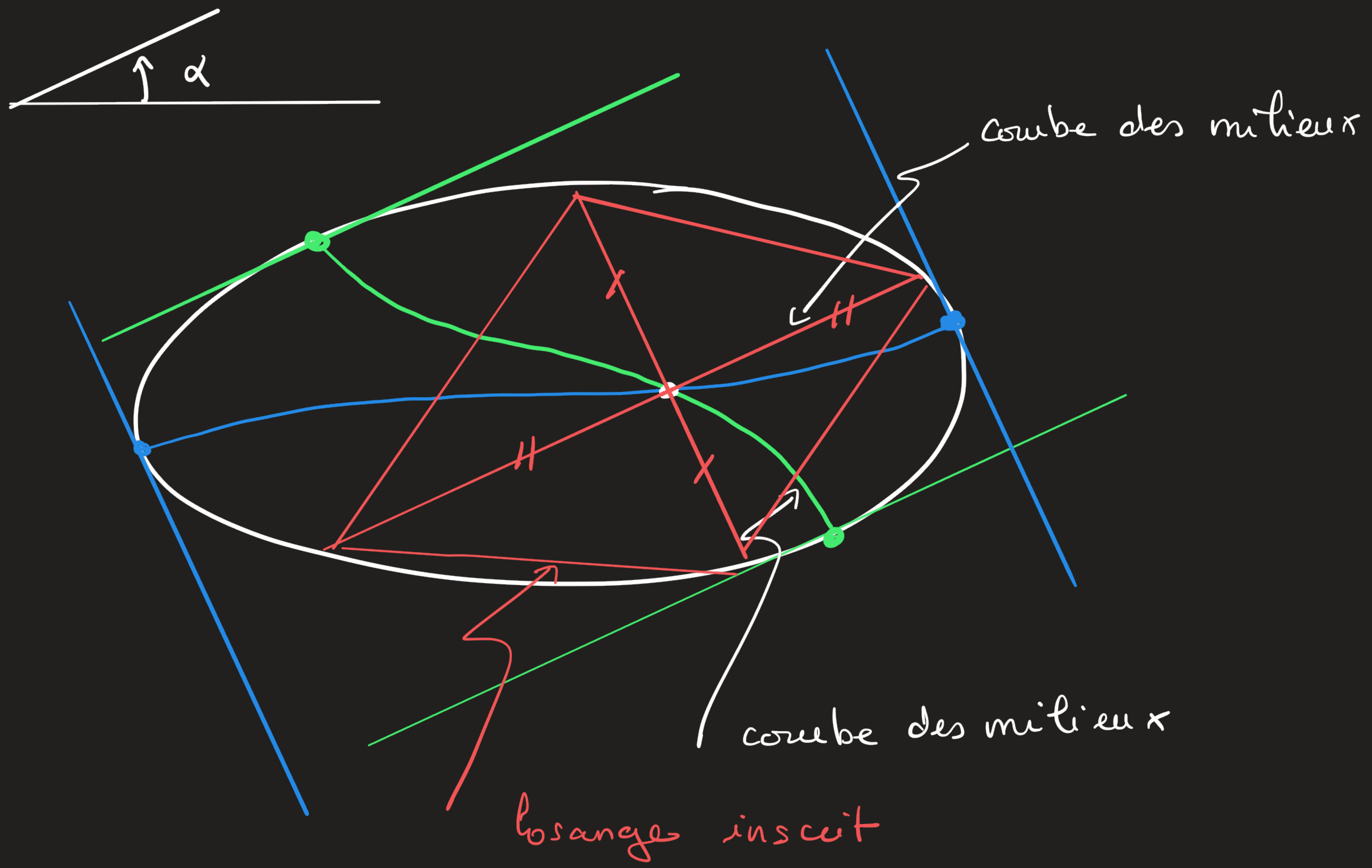


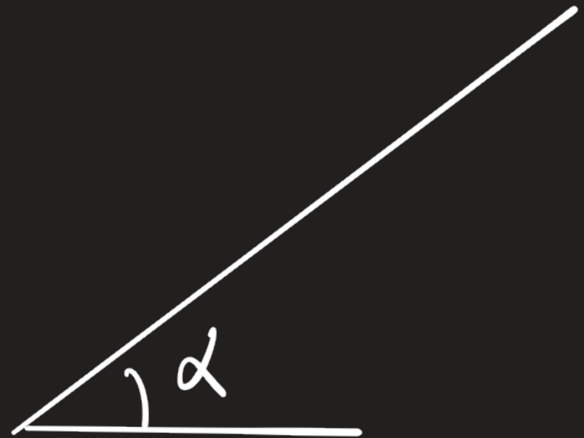

courbe des milieux

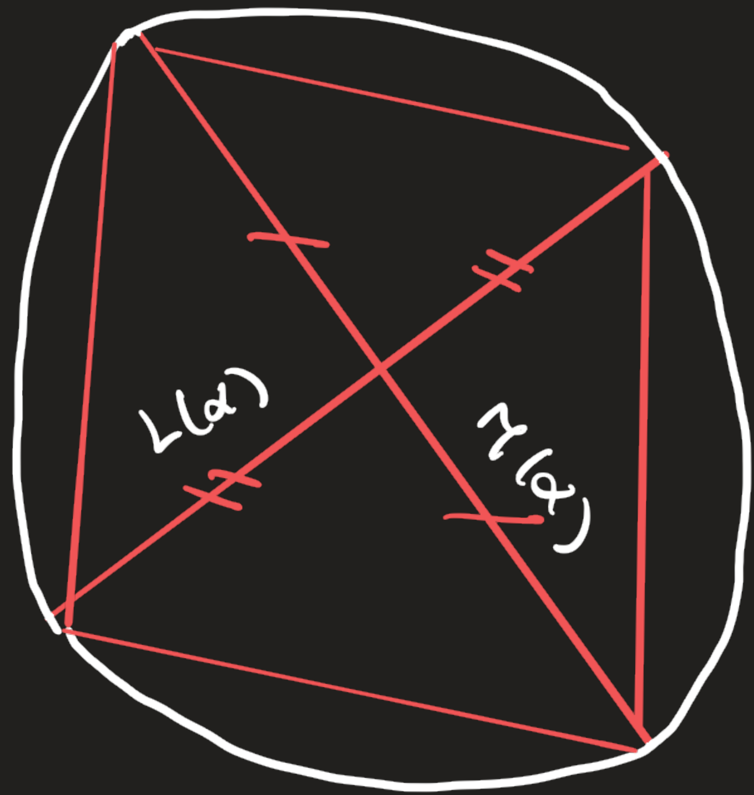

Courbe des milieux



losange dont les
sommets sont sur
la courbe


α
courbe des milieux
courbe des milieux
losange inscrit





$L(\alpha)$ et $M(\alpha)$ sont les mesures des deux diagonales

$$f(\alpha) = L(\alpha) - M(\alpha)$$

$$f(0) = L(0) - M(0)$$

$$f\left(\frac{\pi}{2}\right) = L\left(\frac{\pi}{2}\right) - M\left(\frac{\pi}{2}\right) = M(0) - L(0) = -f(0)$$

Problème du triangle equilatéral inscrit.