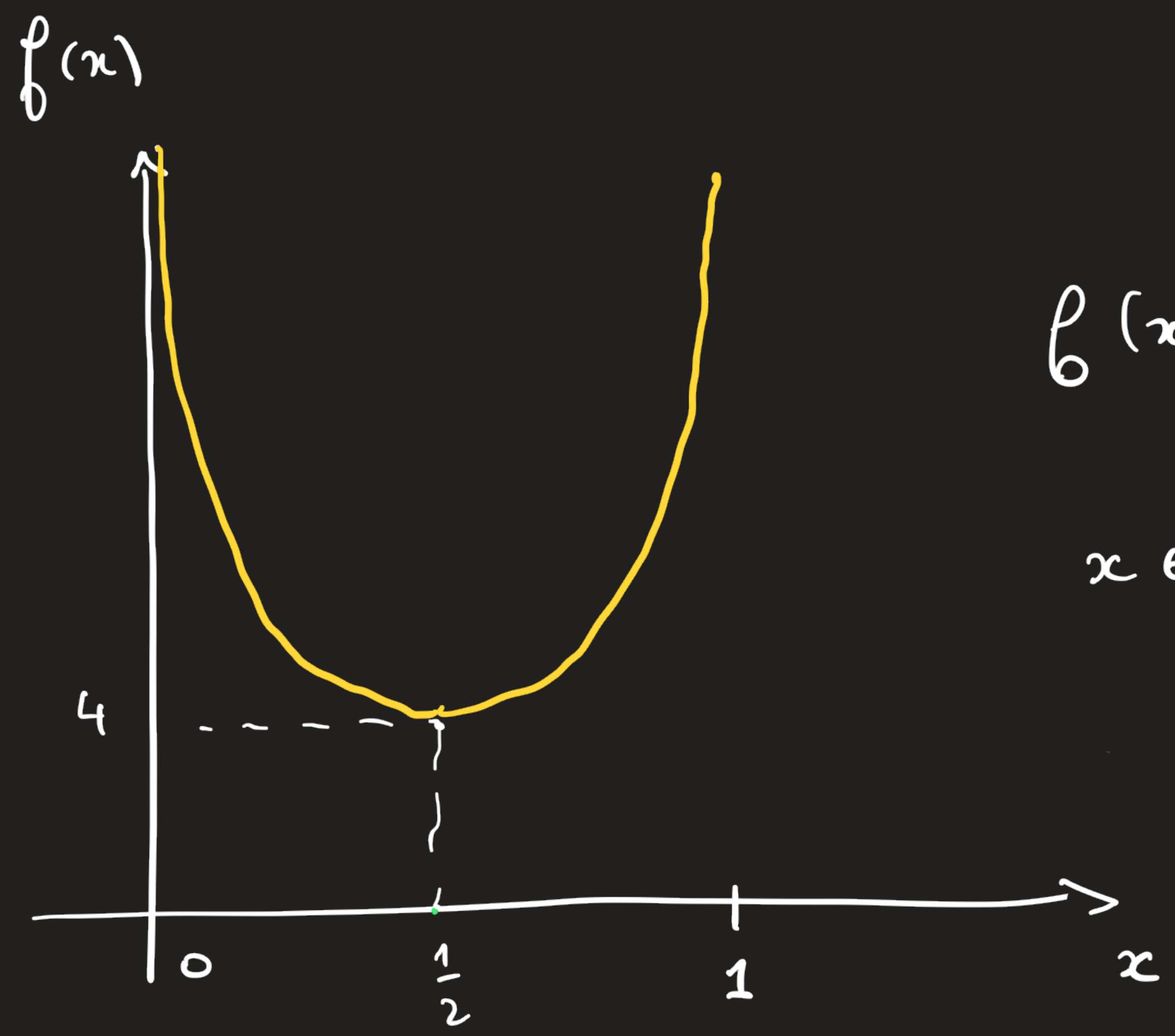
Théorème de la borne atteinte :

$$f : [a,b] \to \mathbb{R} \quad \text{continue}$$

$$\exists x \in [a,b] \qquad f(x) = \max_{[a,b]} f < +\infty$$

Deux hypothèses :
- $[a,b]$ fermé, borné
- f continue

Deux conclusions :
- f est bornée
- le max de f est atteint.

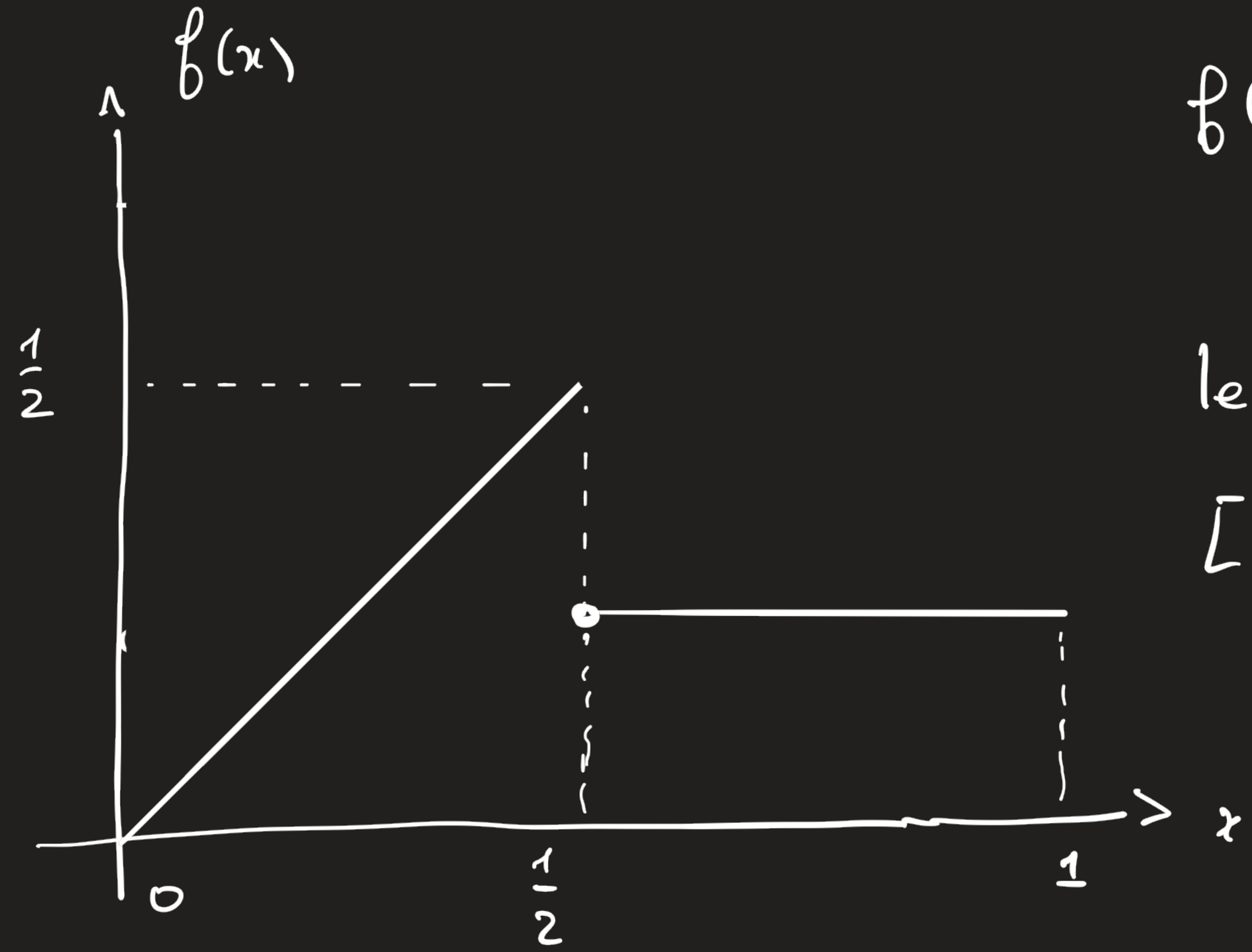


$$f(x) = \frac{1}{x(1-x)}$$

$x \in]0, 1[$

f est continue mais

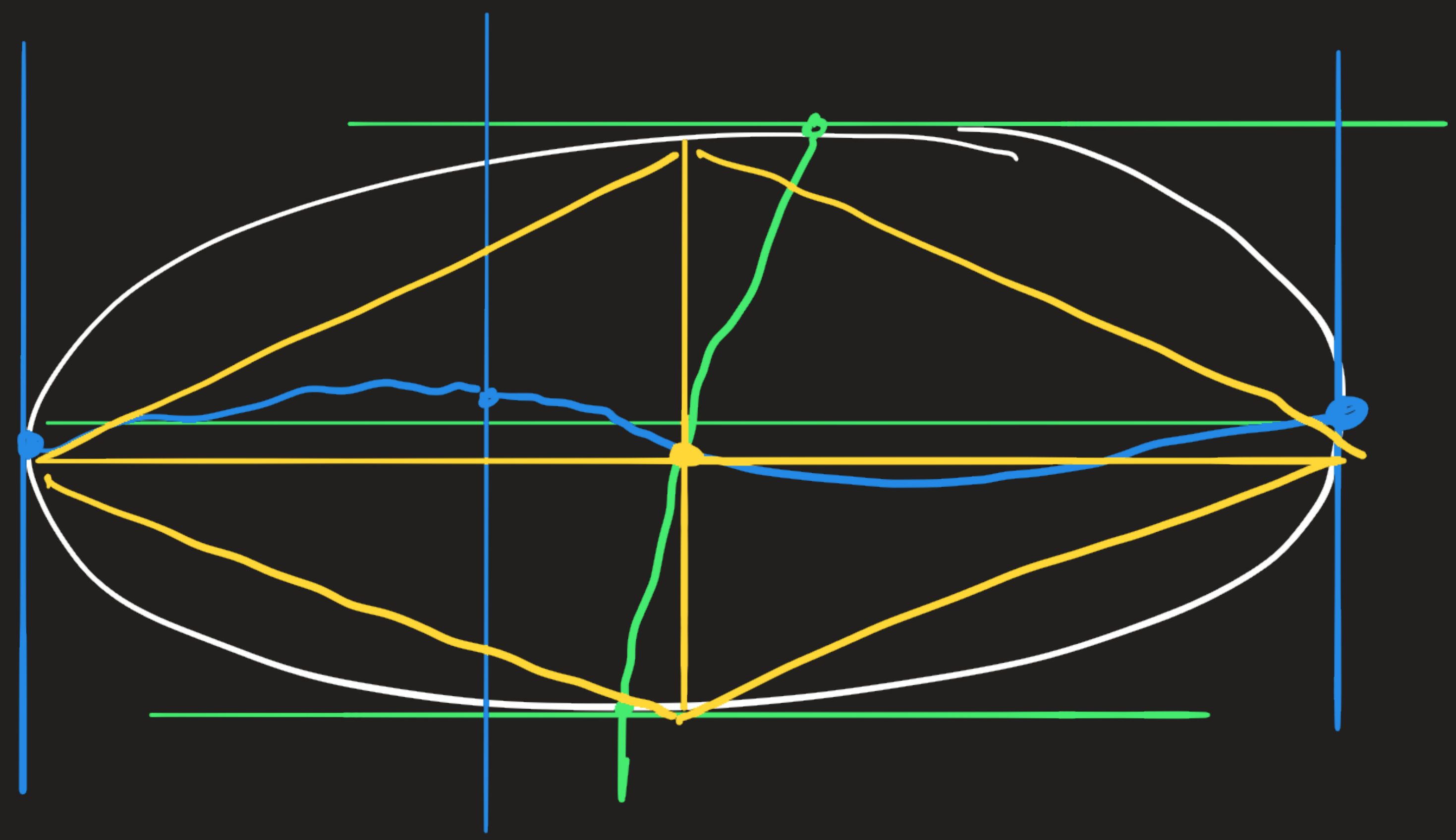
f n'est pas bornée

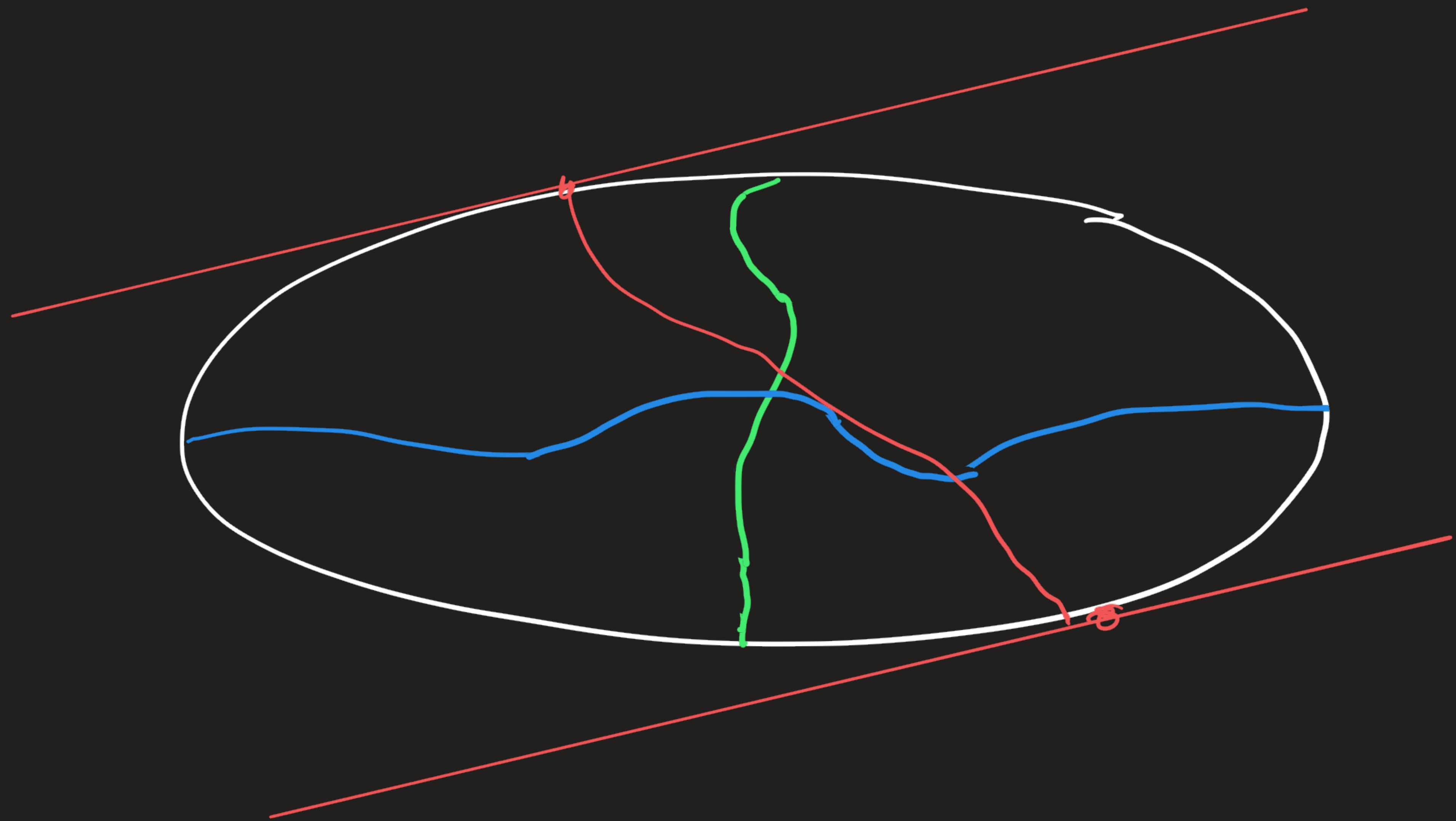


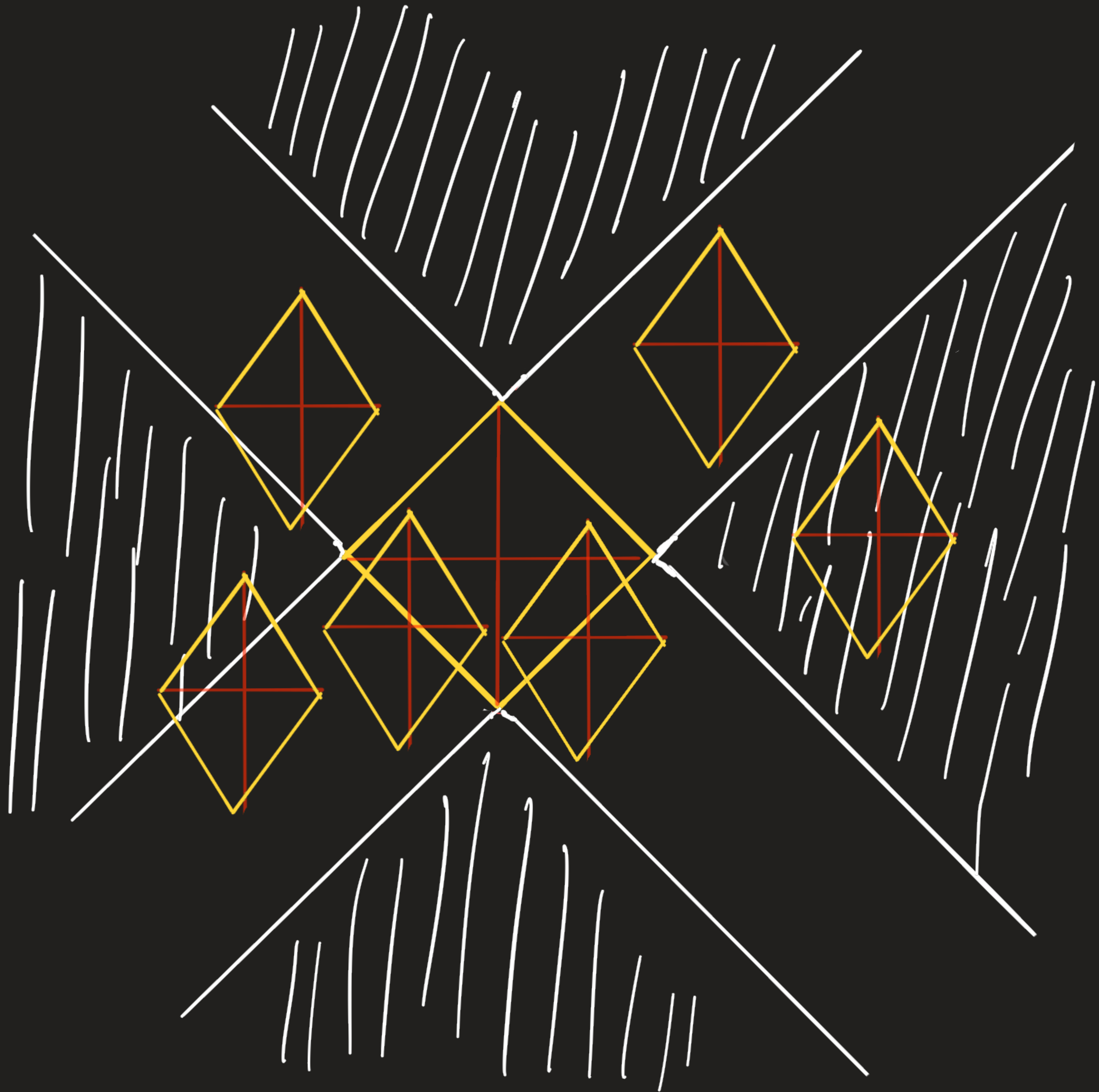
$$f(x) = \begin{cases} x & \text{si } x < \frac{1}{2} \\ \frac{1}{4} & \text{si } x \geq \frac{1}{2} \end{cases}$$

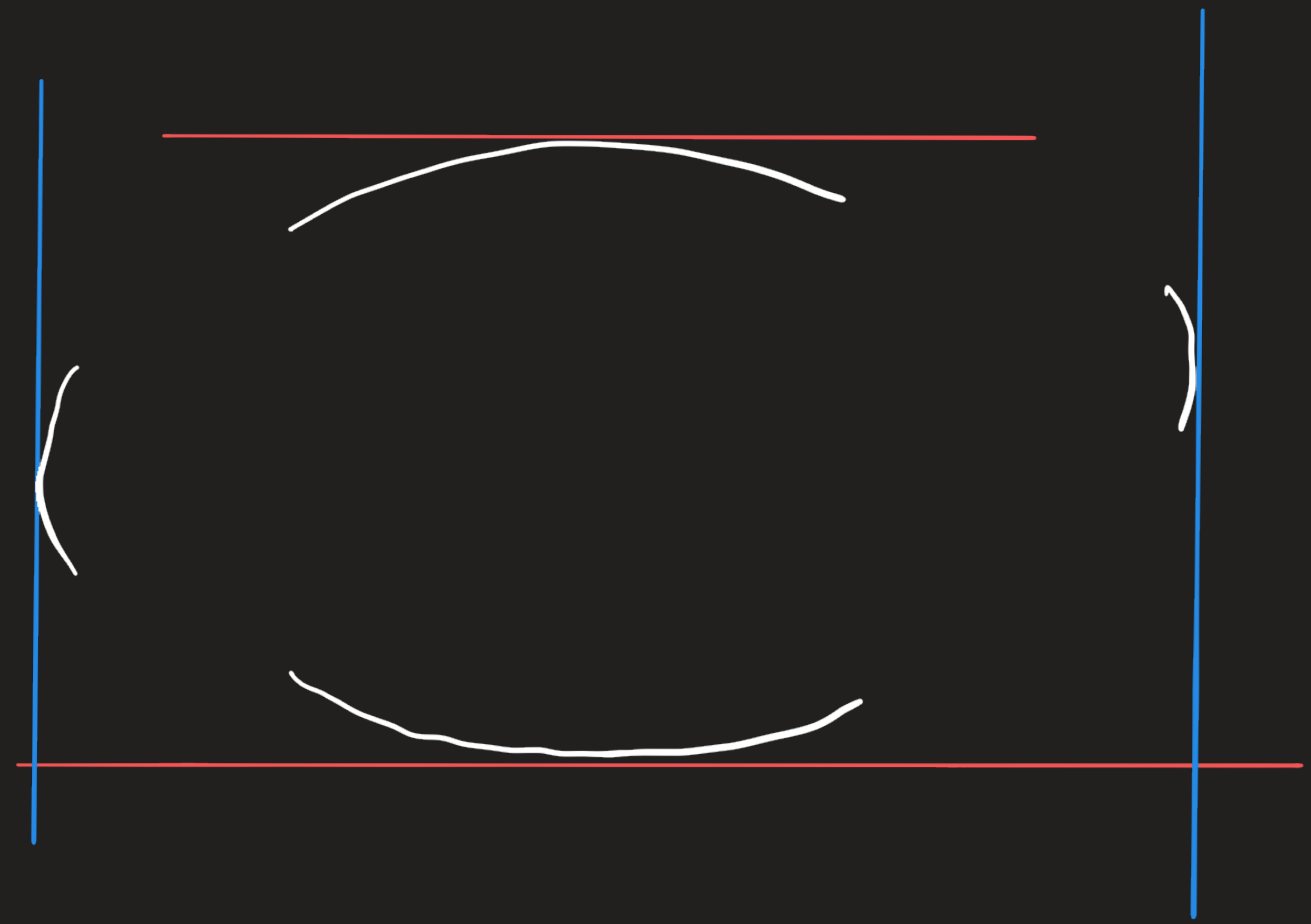
le sup de f sur $[0,1]$ est $1/2$, il n'est pas atteint.

Retour sur le cours d'hier



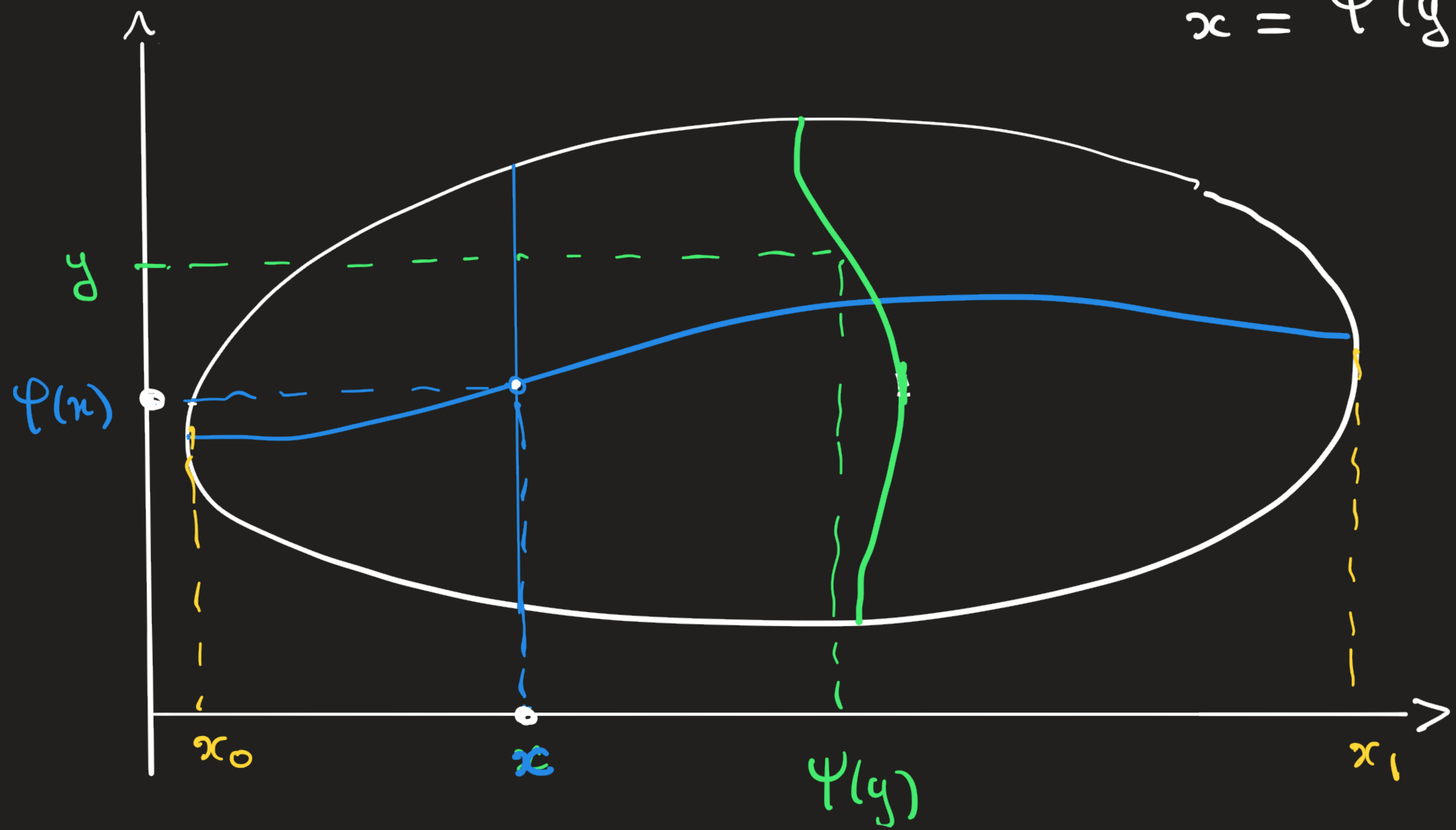






$$y = \varphi(x)$$

$$x = \psi(y)$$

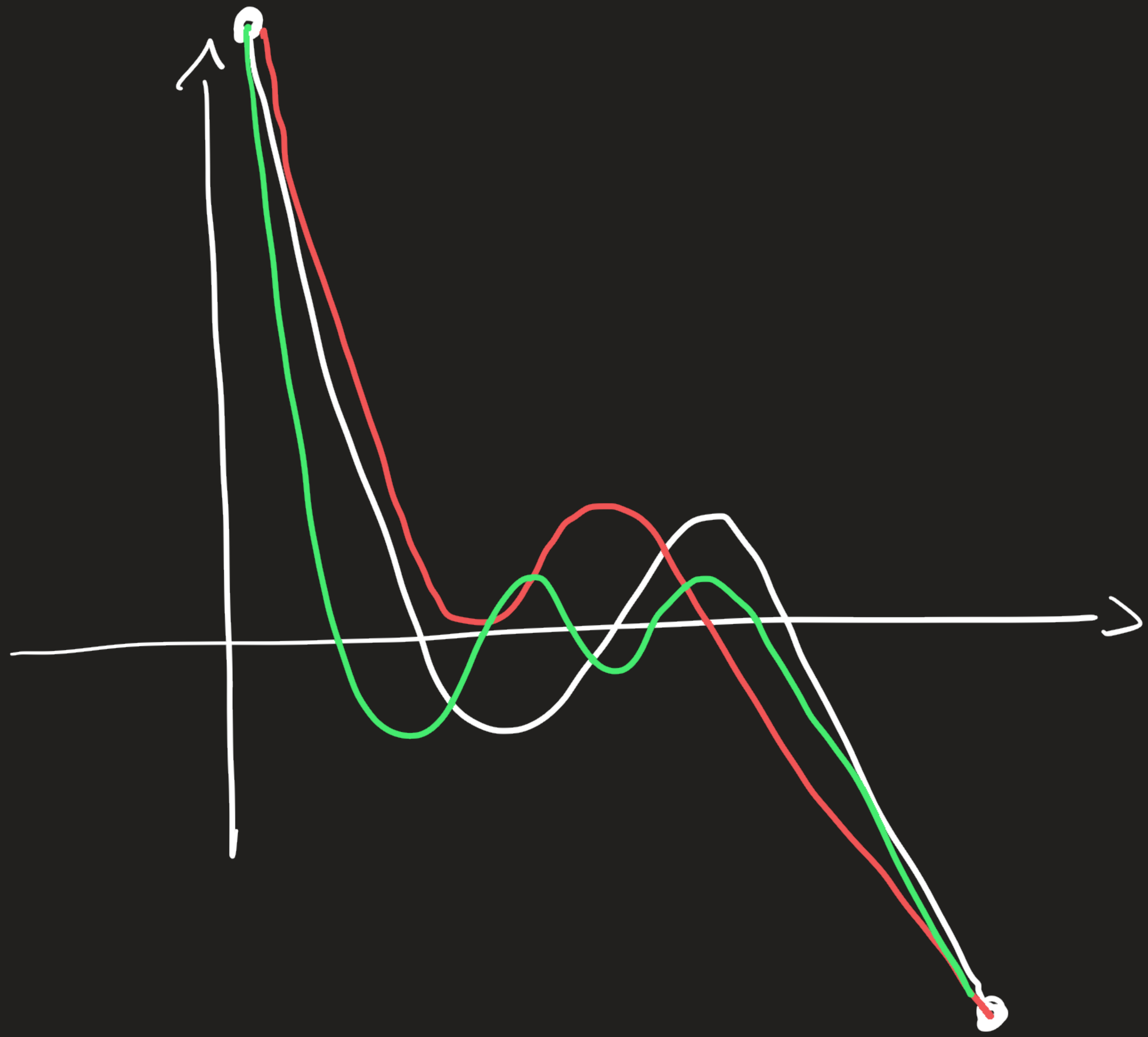


$$g(x) = x - \psi(\varphi(x))$$

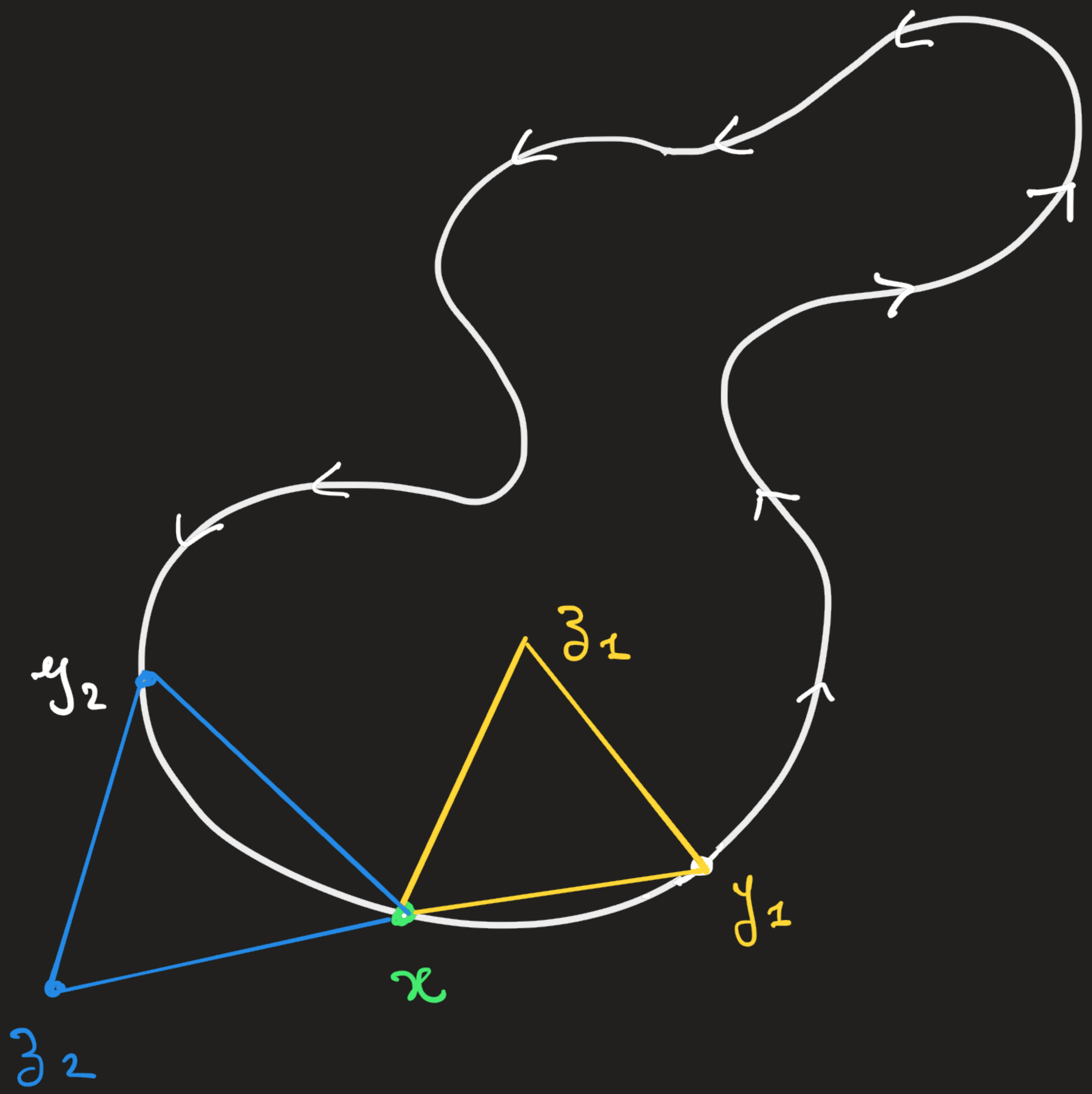
$g(x_0) < 0$ $\qquad$ $g(x_1) > 0$

$$x = \psi(\varphi(x))$$

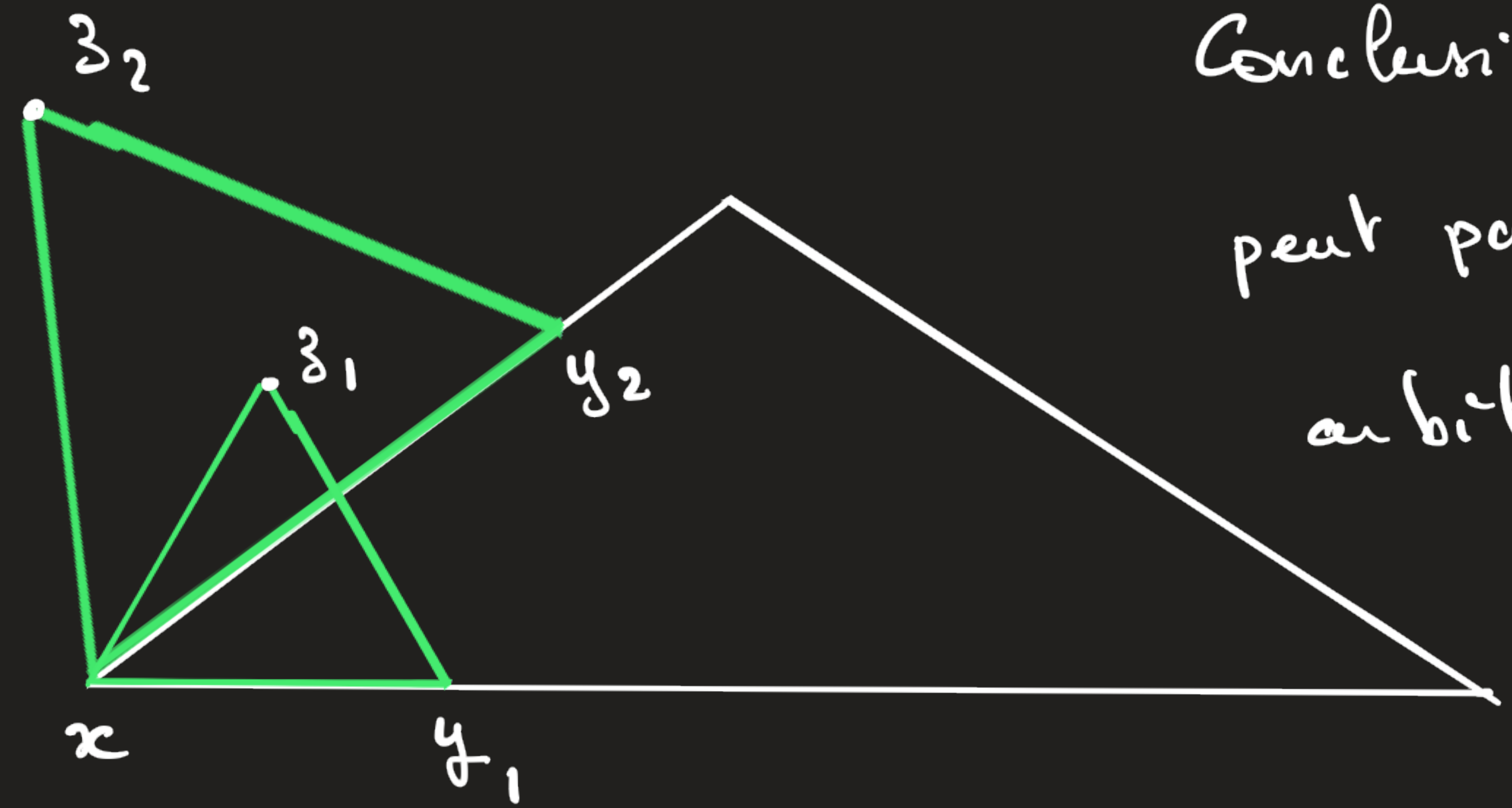
$$y = \varphi(x)$$



Théorème (M. Meyerson 1982). Soit Γ une courbe de Jordan. Tout triangle est semblable à un triangle dont les trois sommets sont des points de Γ.

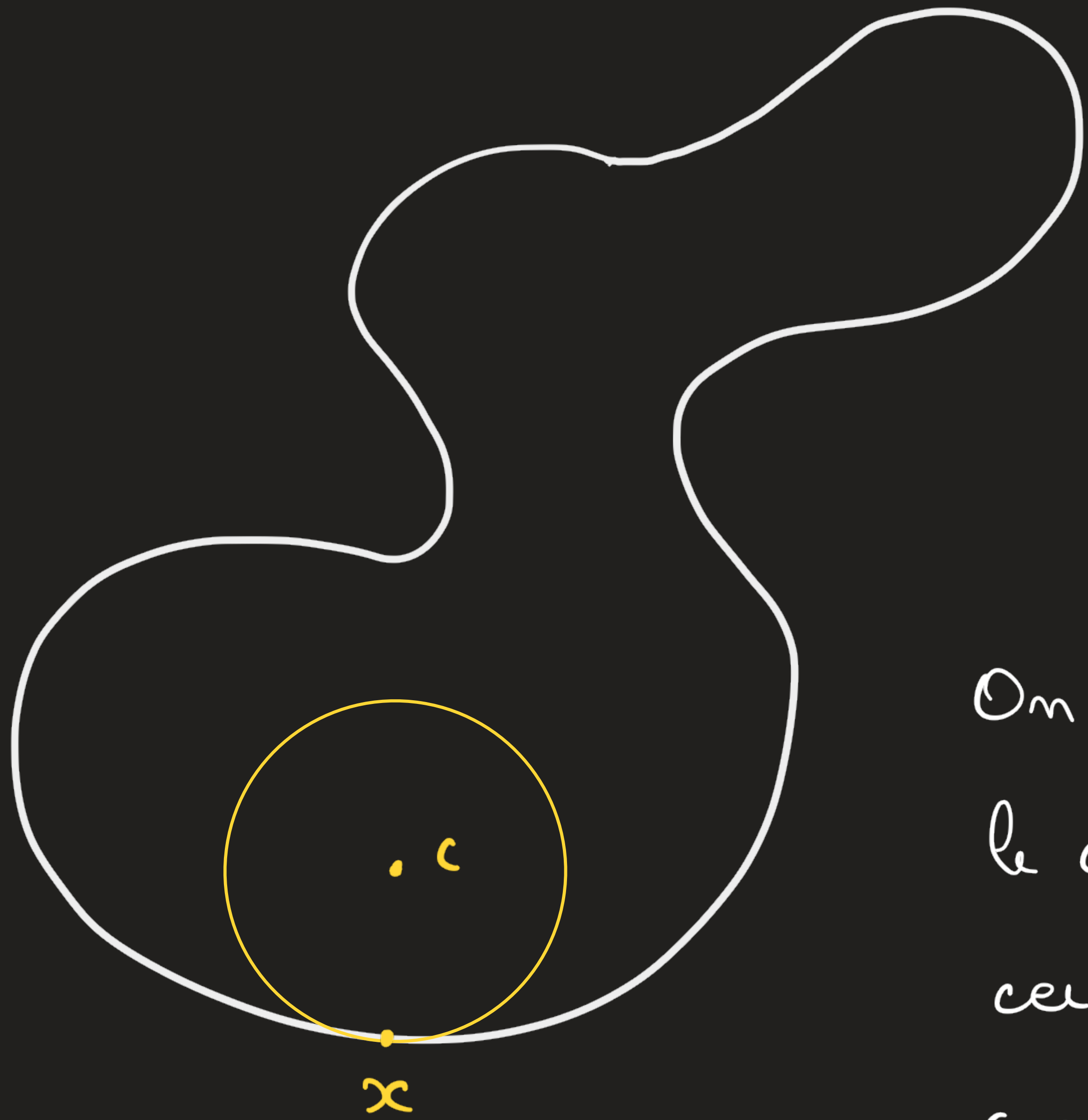


Une très bonne idée !

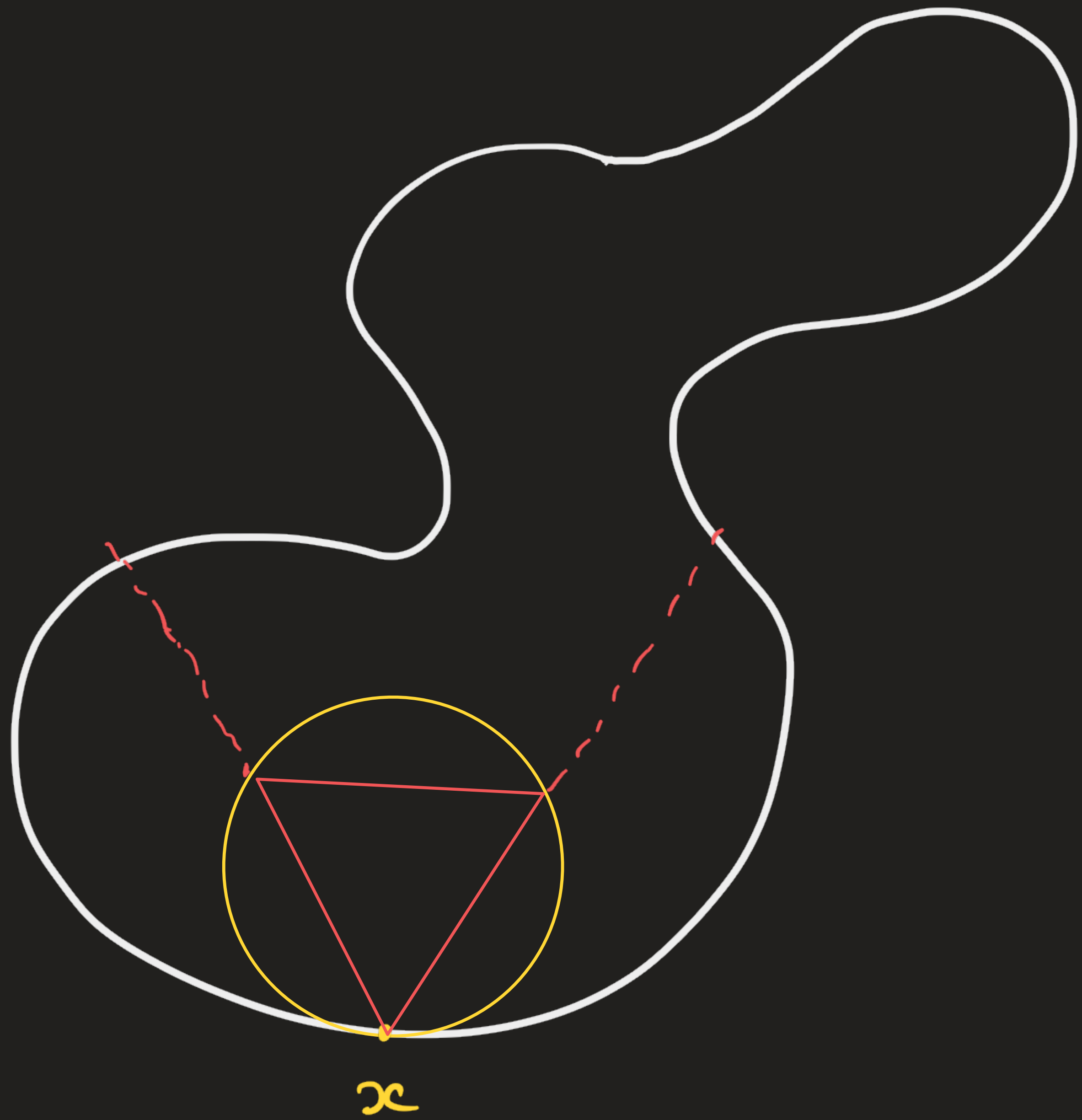


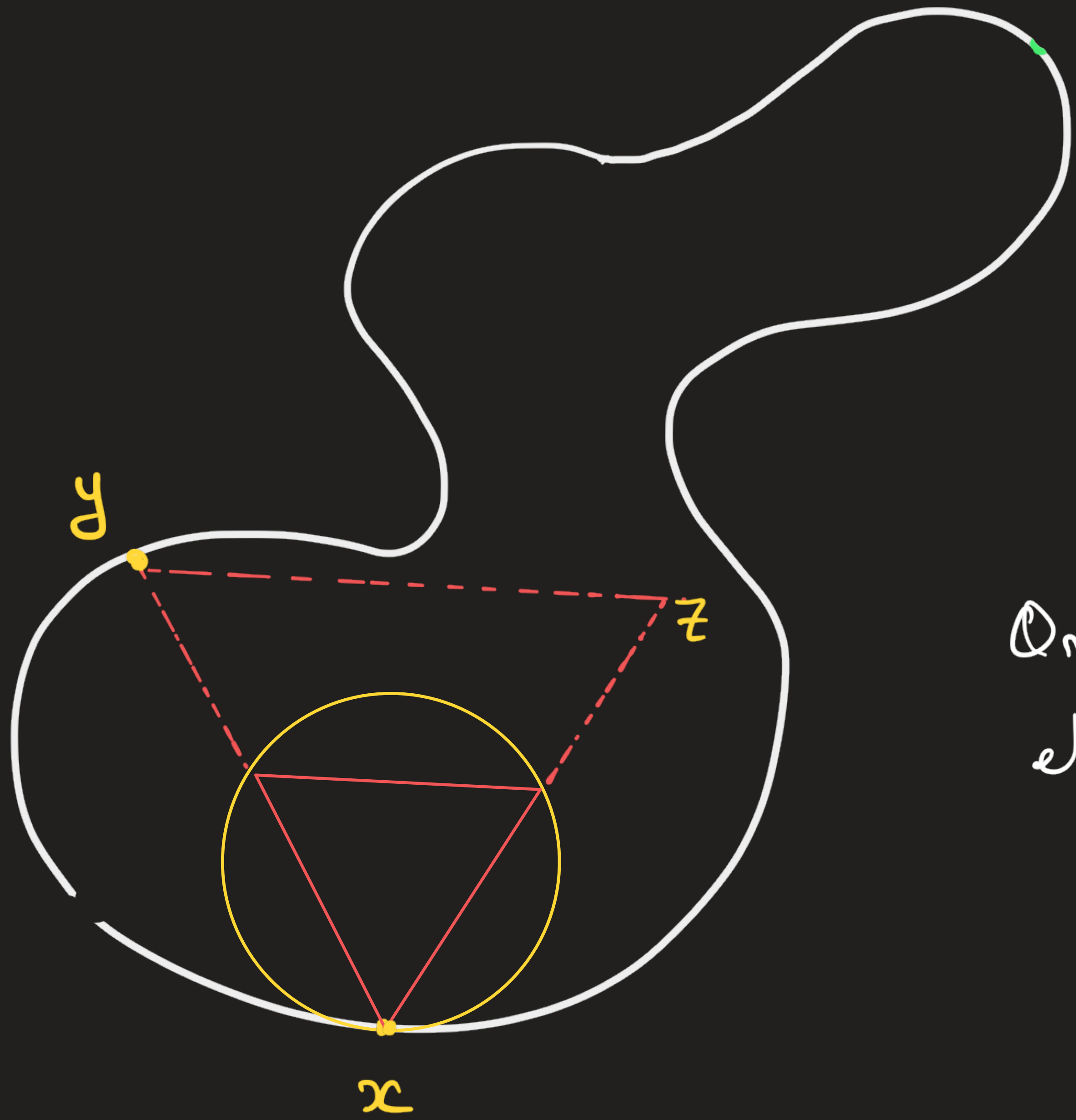
Un contre-exemple !

Conclusion : x ne peut pas être choisi arbitrairement.

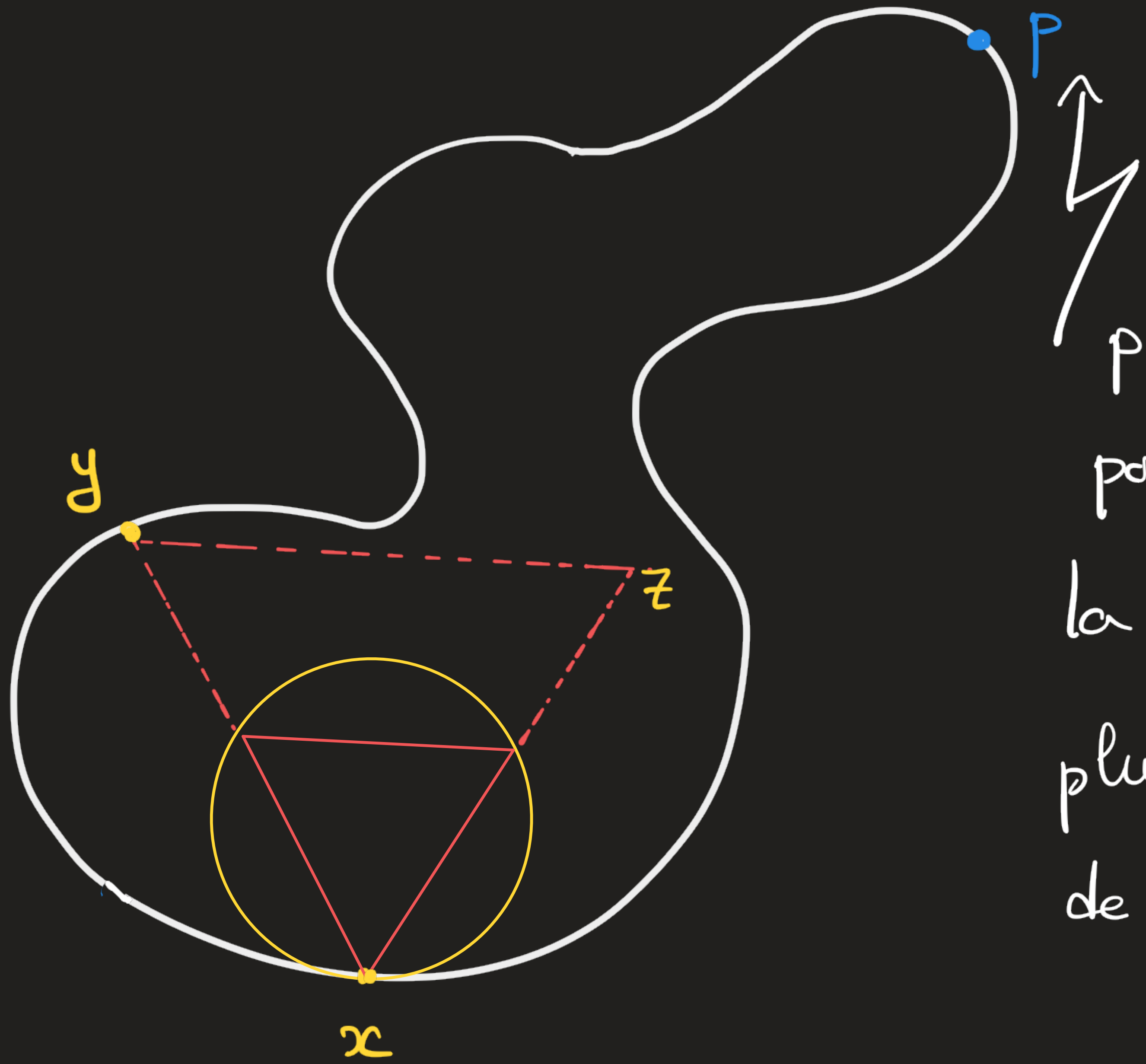


On choisit c le centre du cercle et on construit x

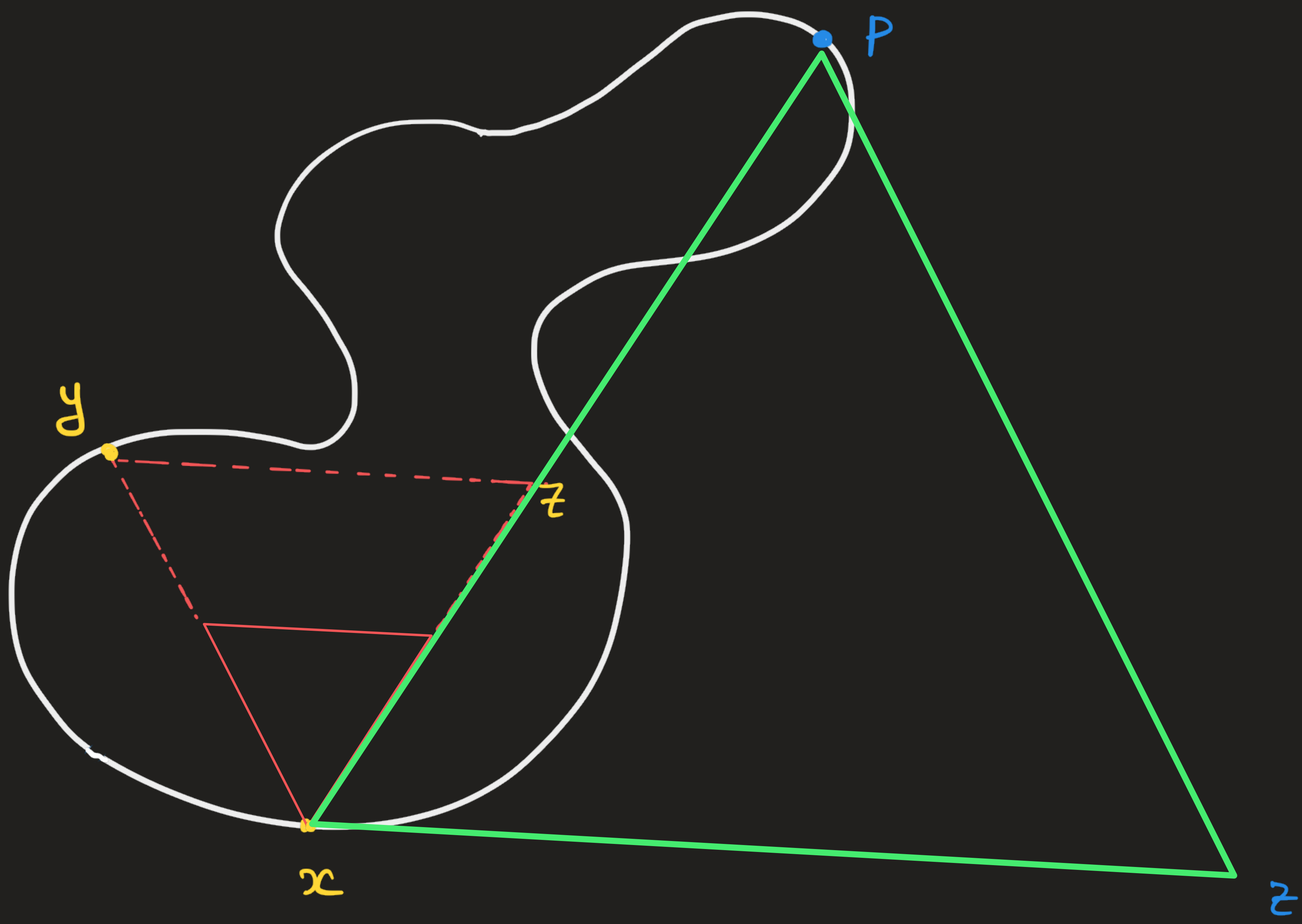

x



On construit y et z



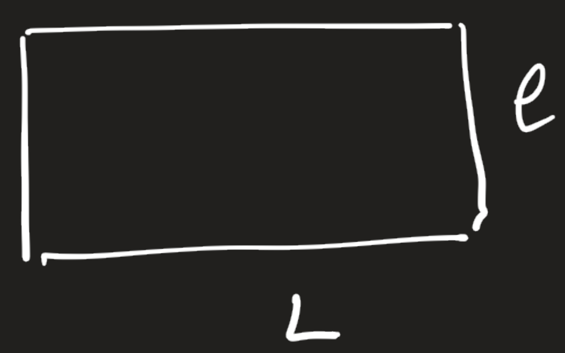
p est le point de la courbe le plus éloigné de x



P
y
z
x
z

Théorème (J. Green - A. Lobb 2020) Soit Γ une courbe <u>régulière</u>. Pour tout $r>0$ il existe un rectangle dont les 4 sommets sont des points de Γ et le rapport des côtés est égal à r.

$$\frac{\ell}{L} = r$$

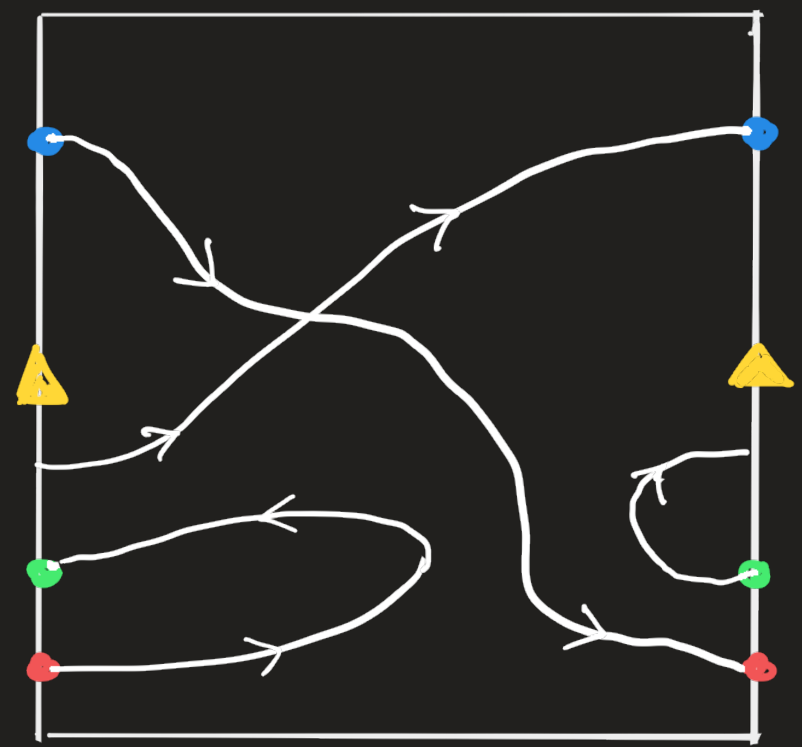


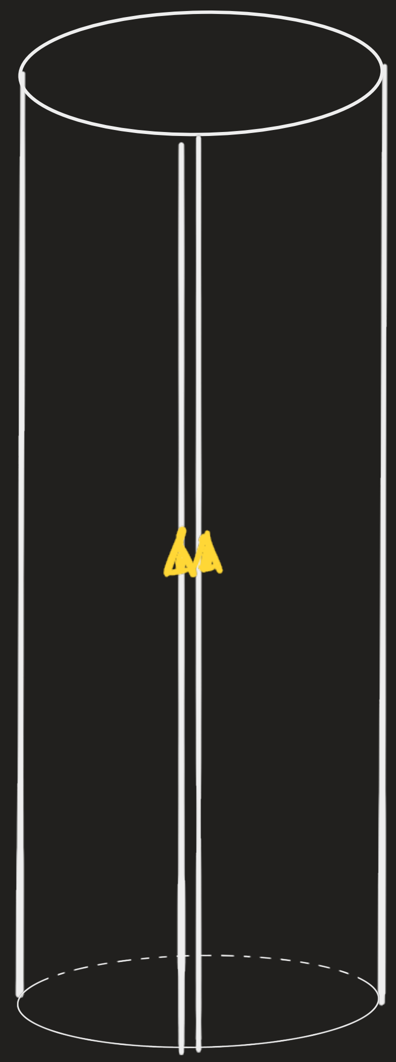
Quelques éléments de la théorie des surfaces

I

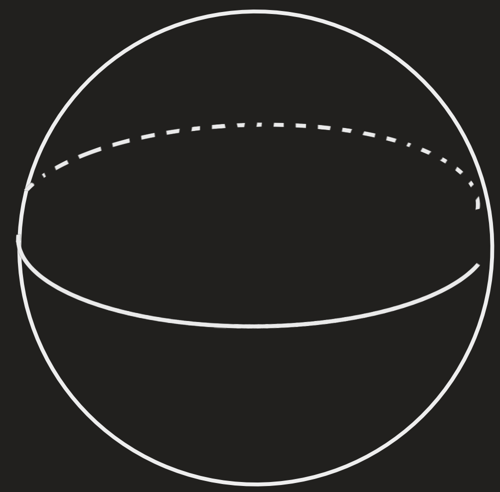
le cylindre

Un chemin sur un cylindre

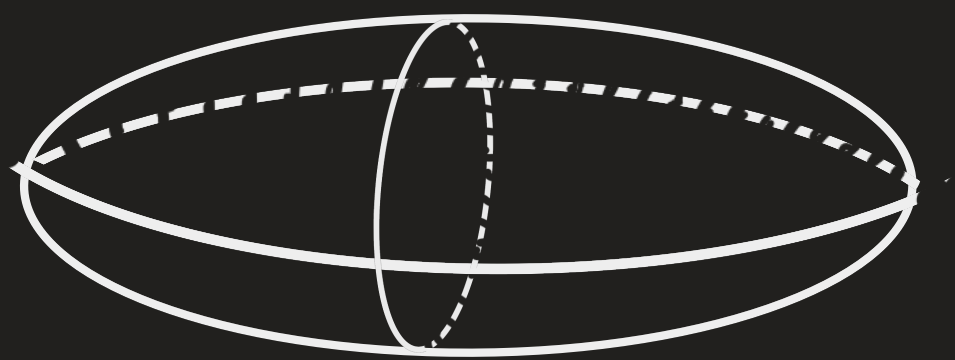




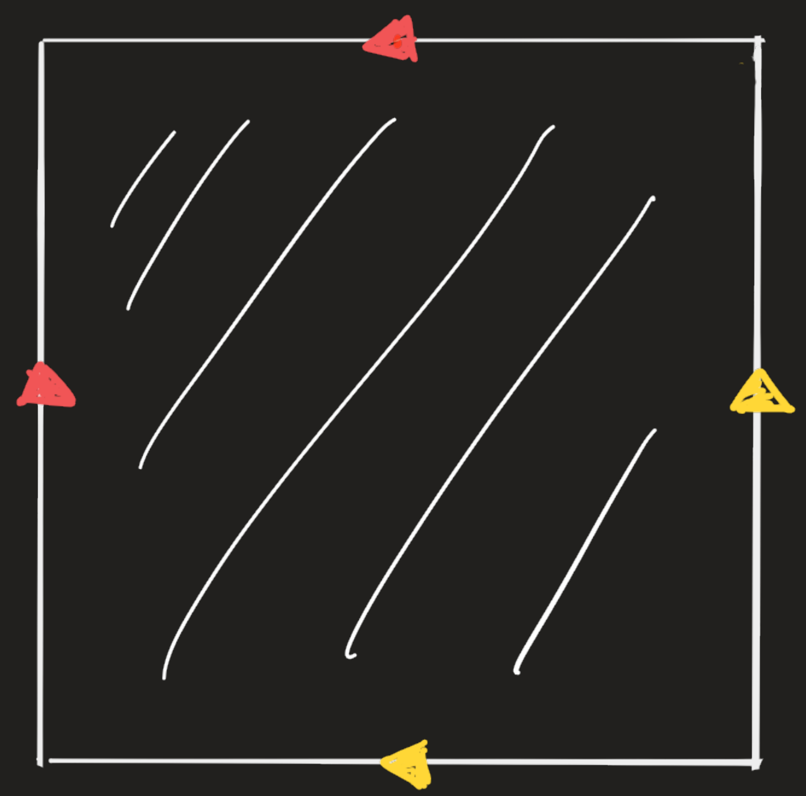
II

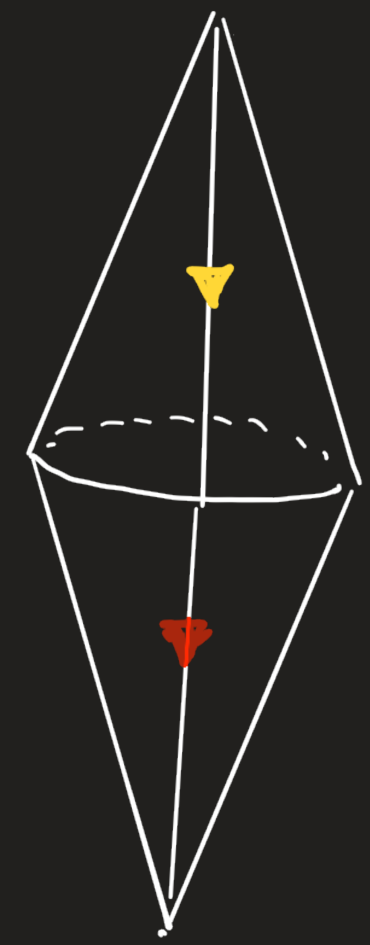


La sphère S^2

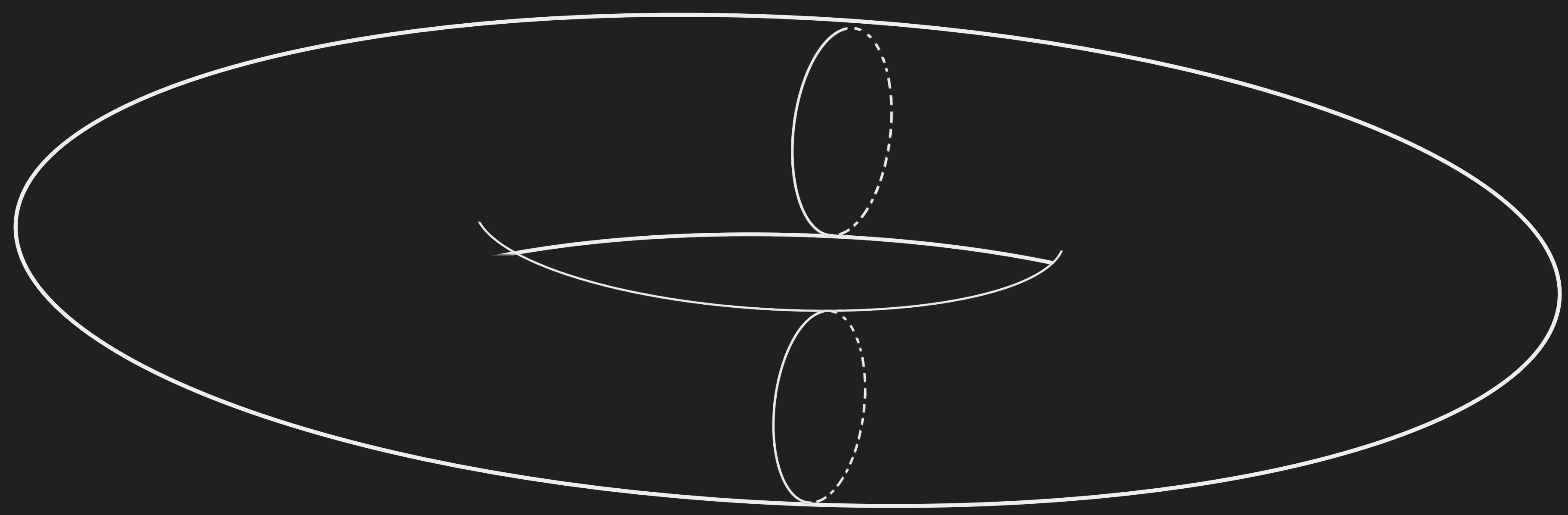


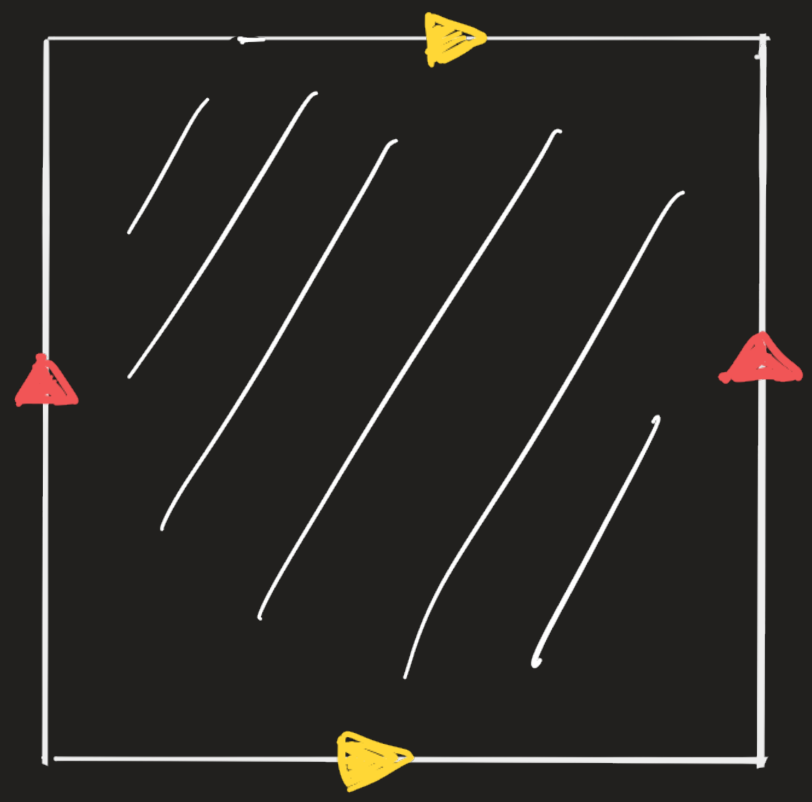
ellipsoïde

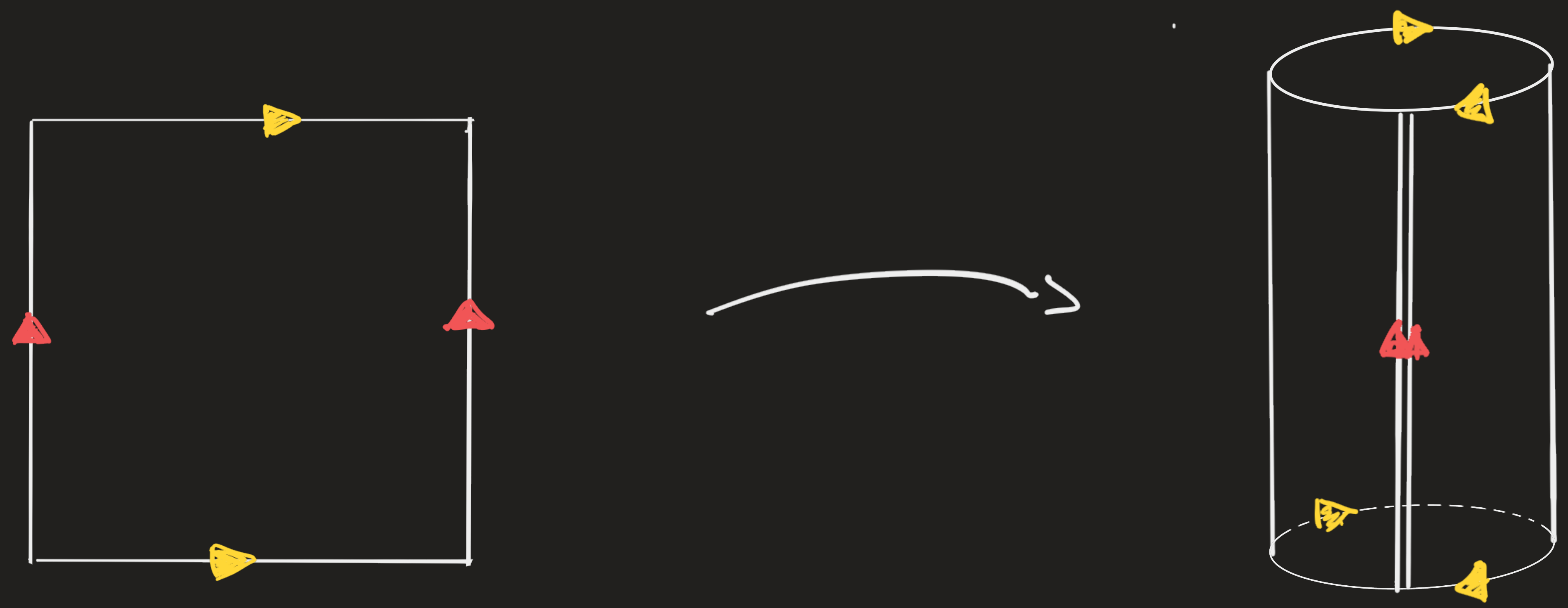


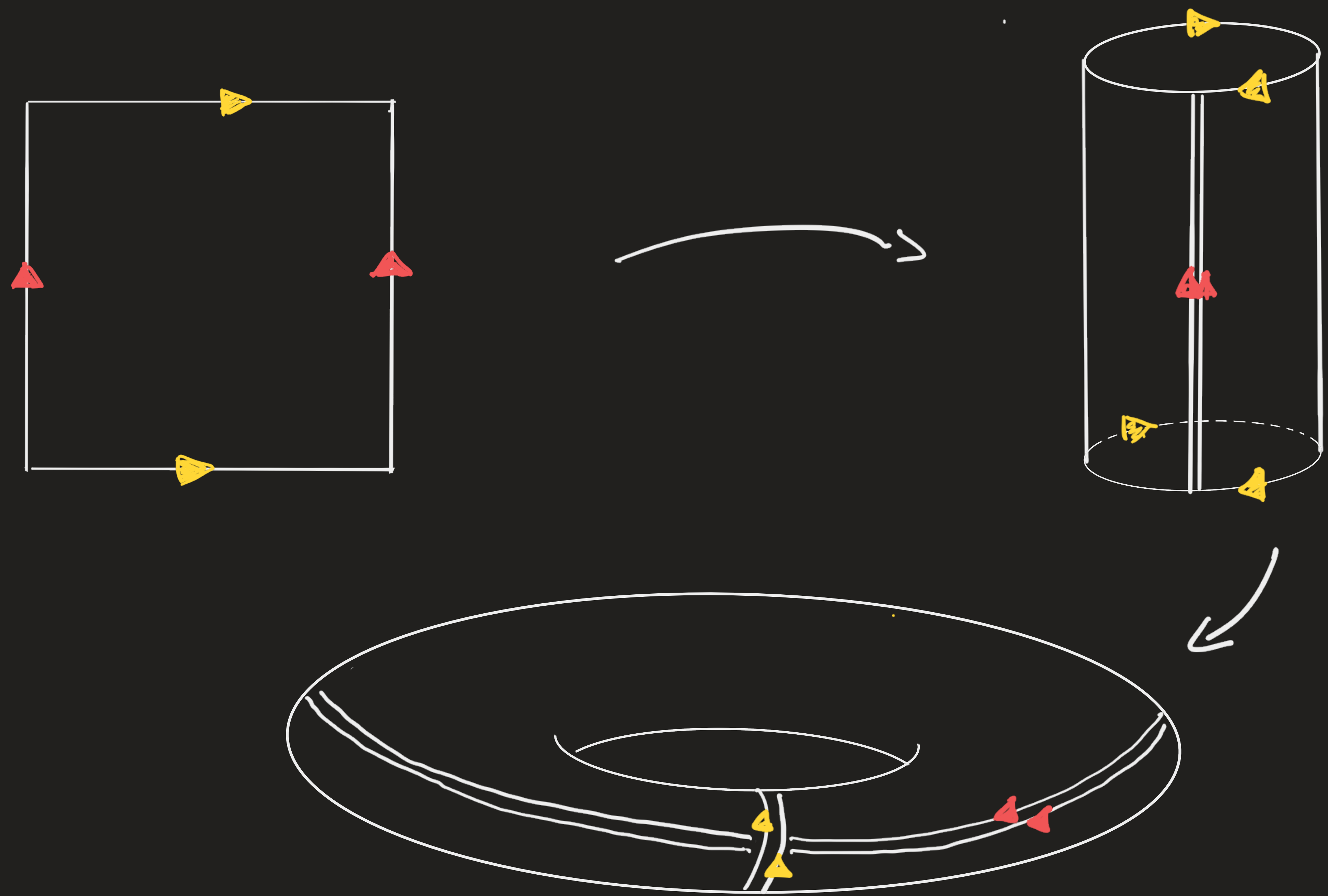


III le tore. T^2

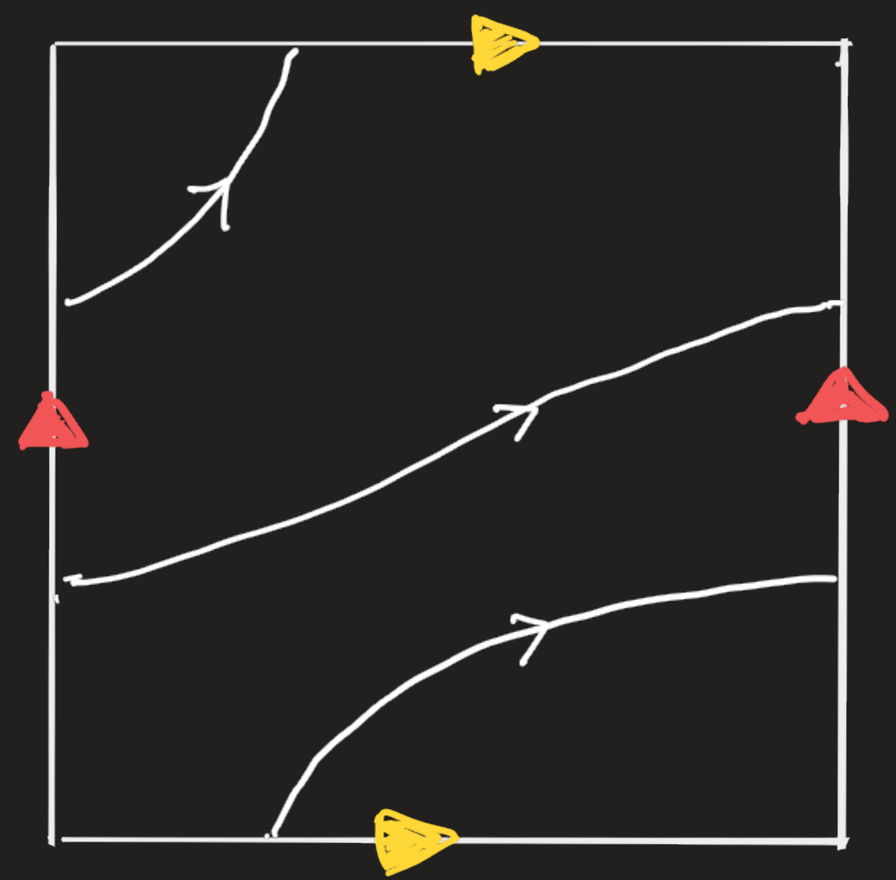




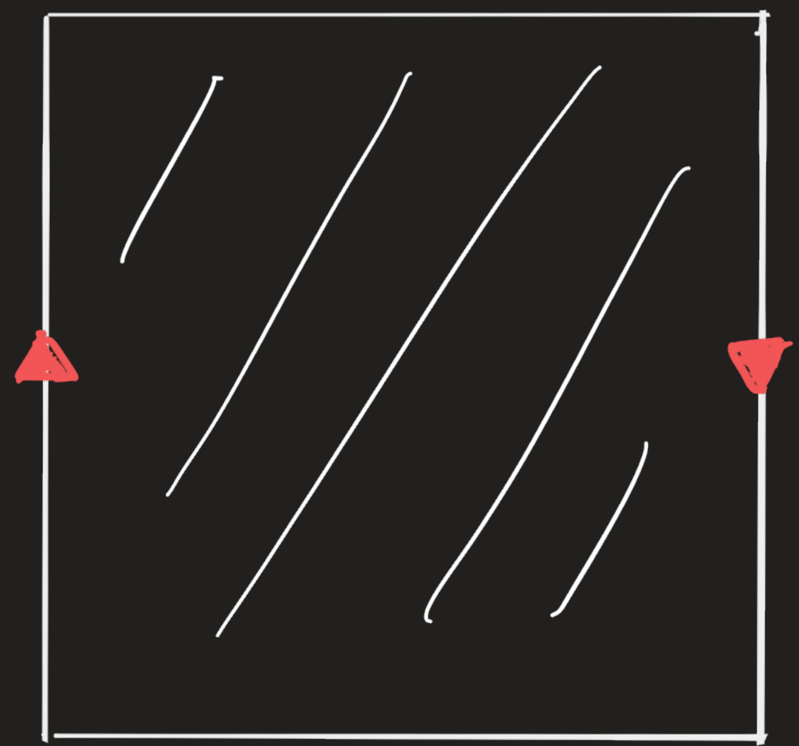




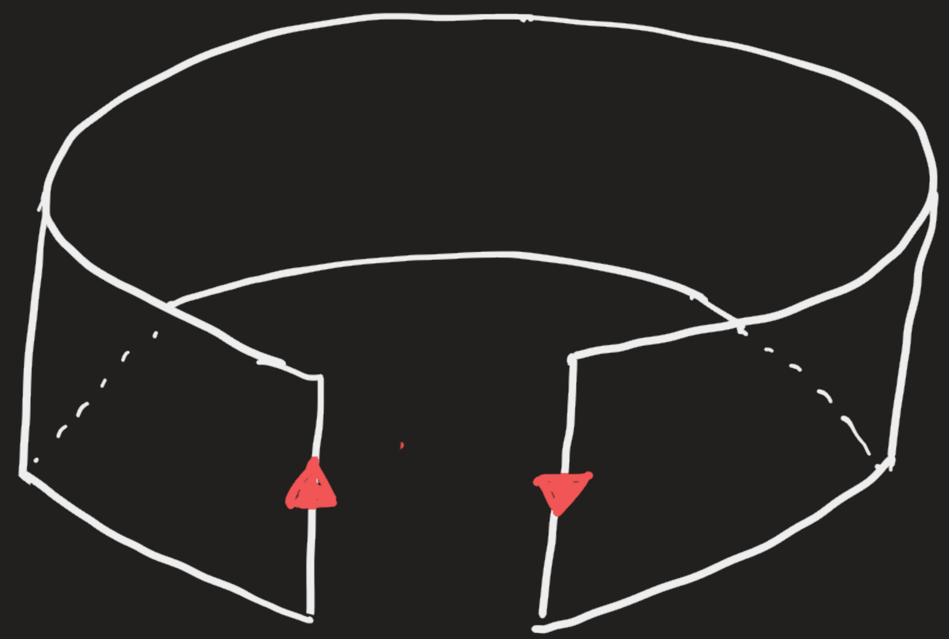
Un chemin sur le tore



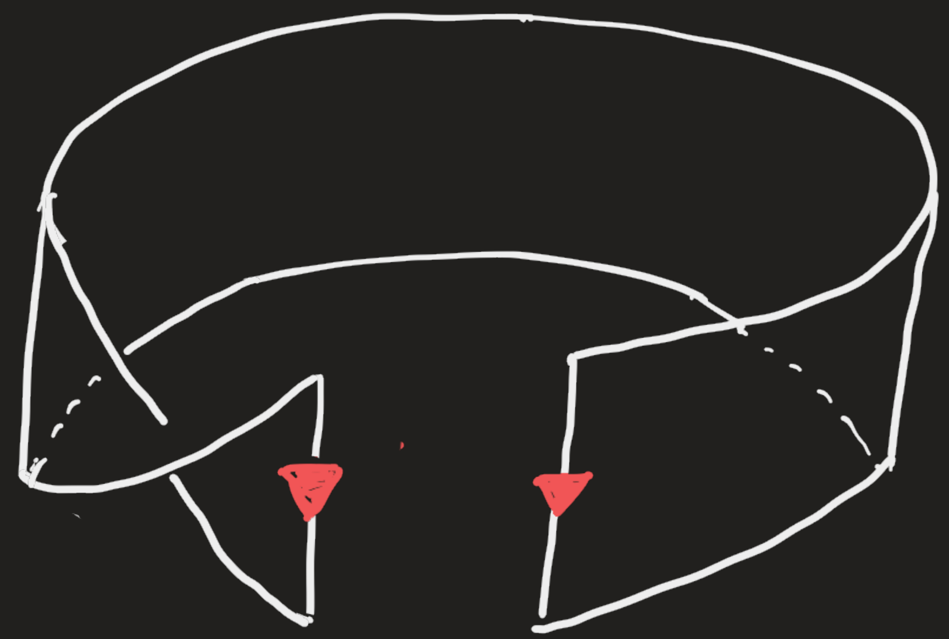
(IV) La bande de Möbius



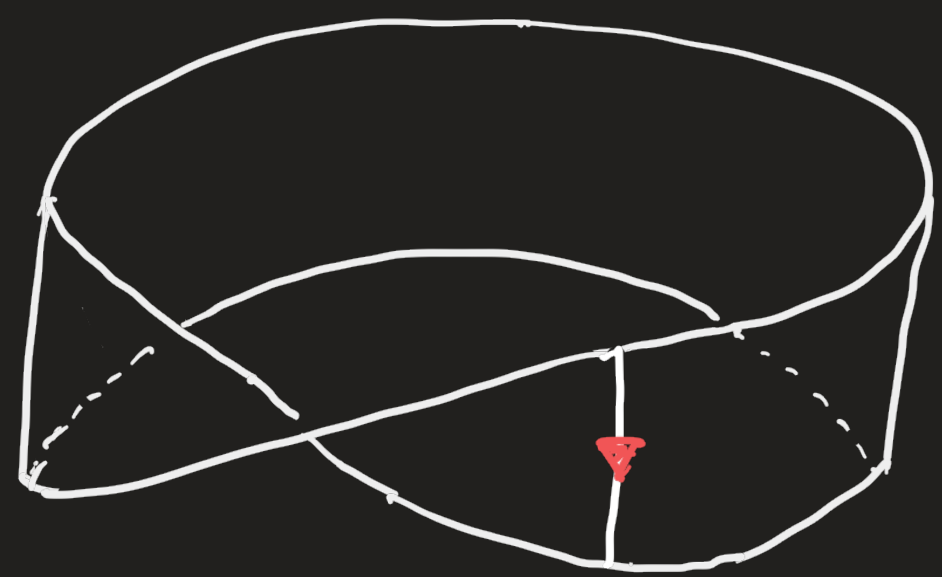
La bande de Möbius



La bande de Möbius

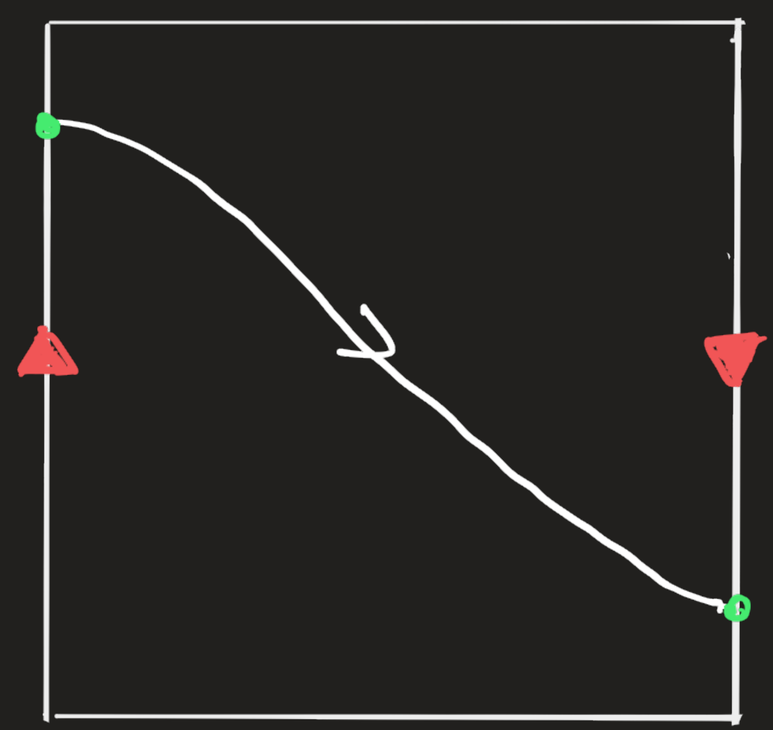


La bande de Möbius n'a qu'un seul bord.

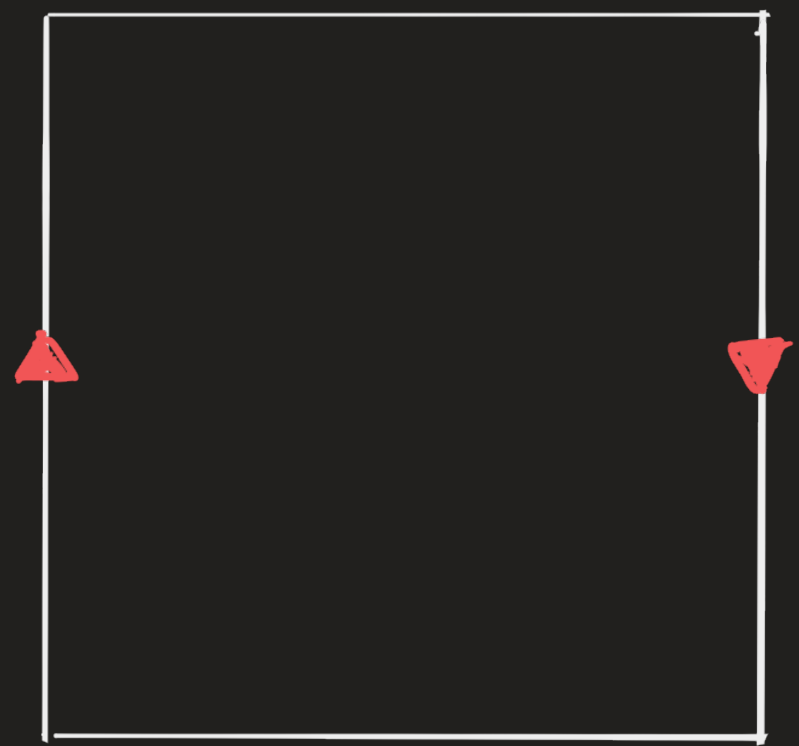


Un chemin sur la bande

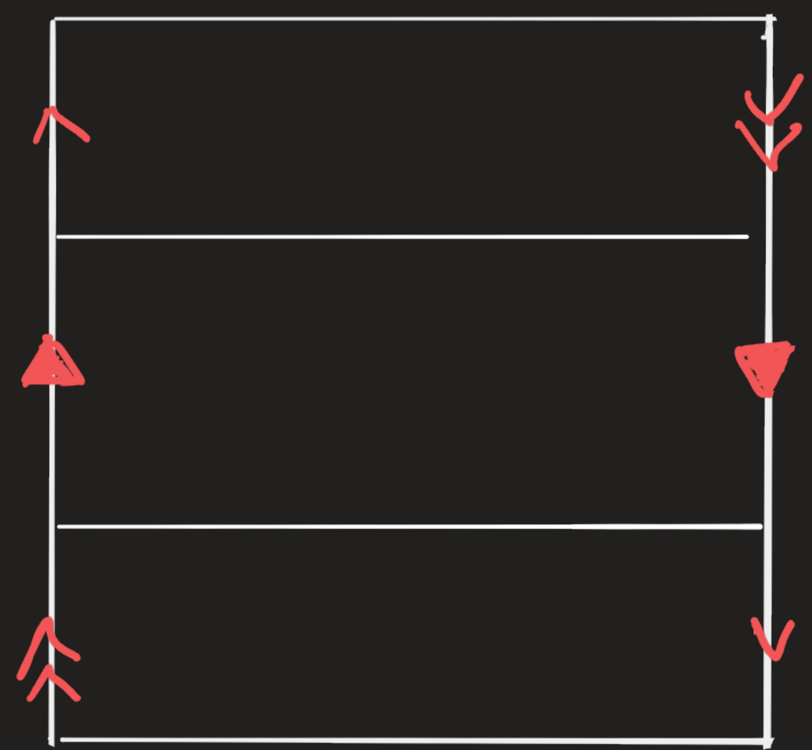
de Möbius



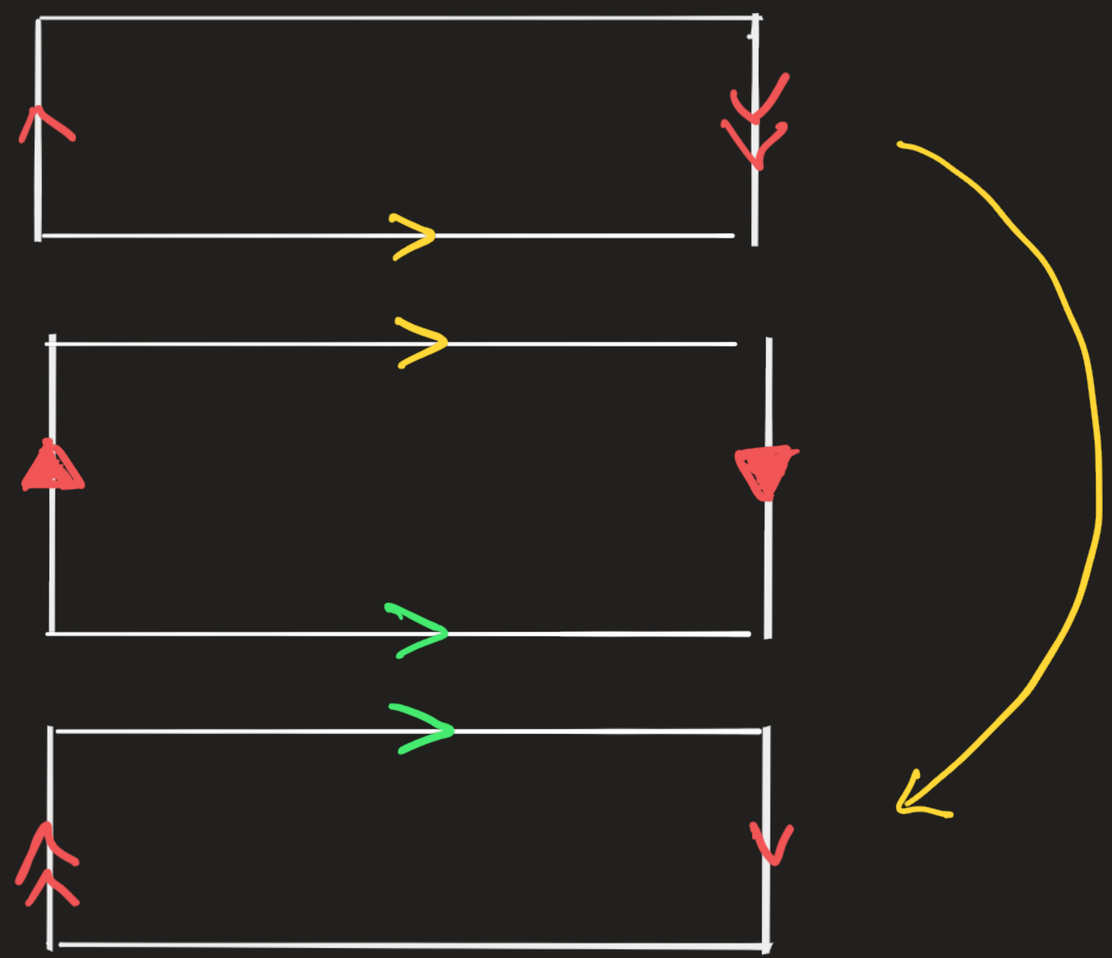
Découper la bande de Möbius



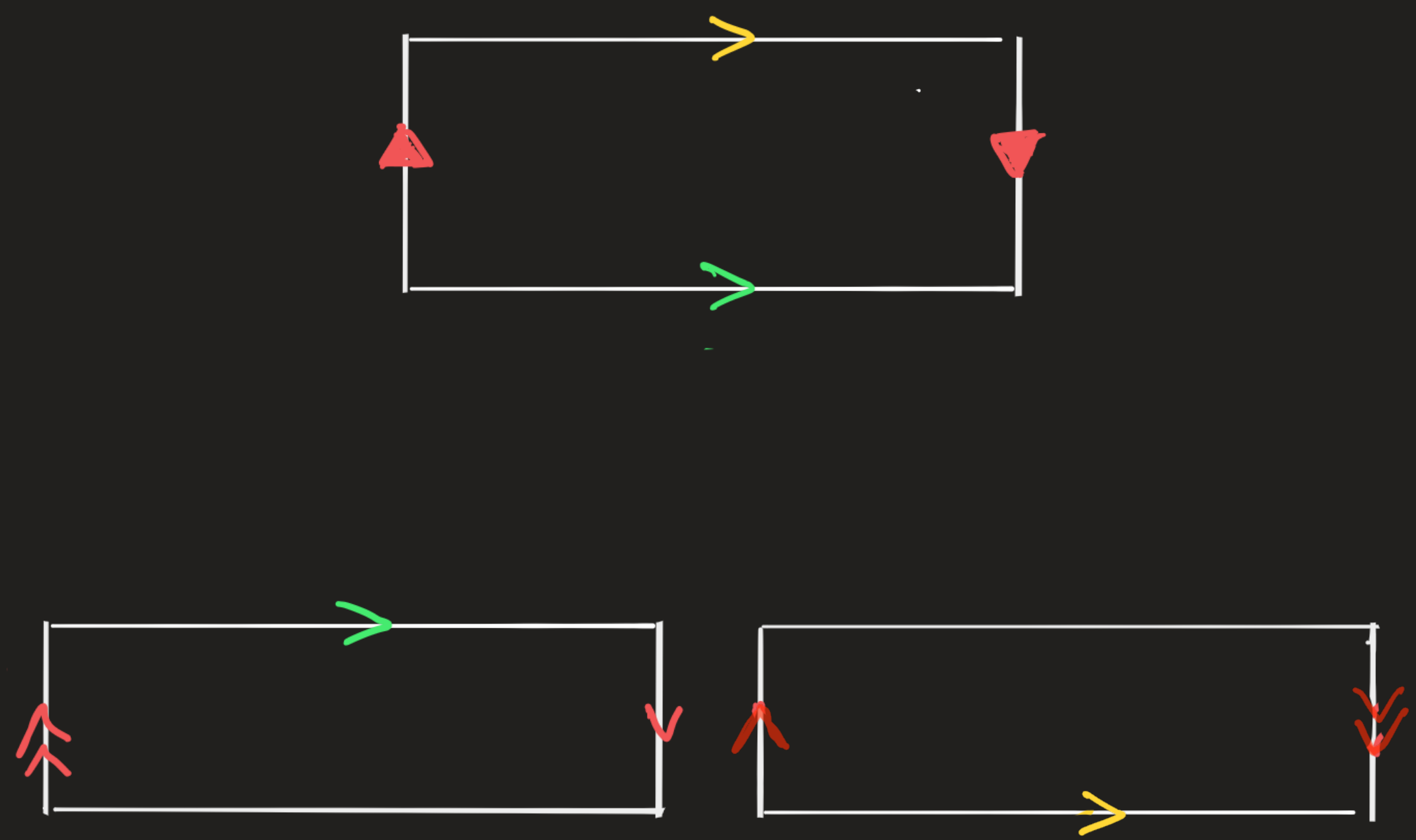
Découper la bande de Möbius



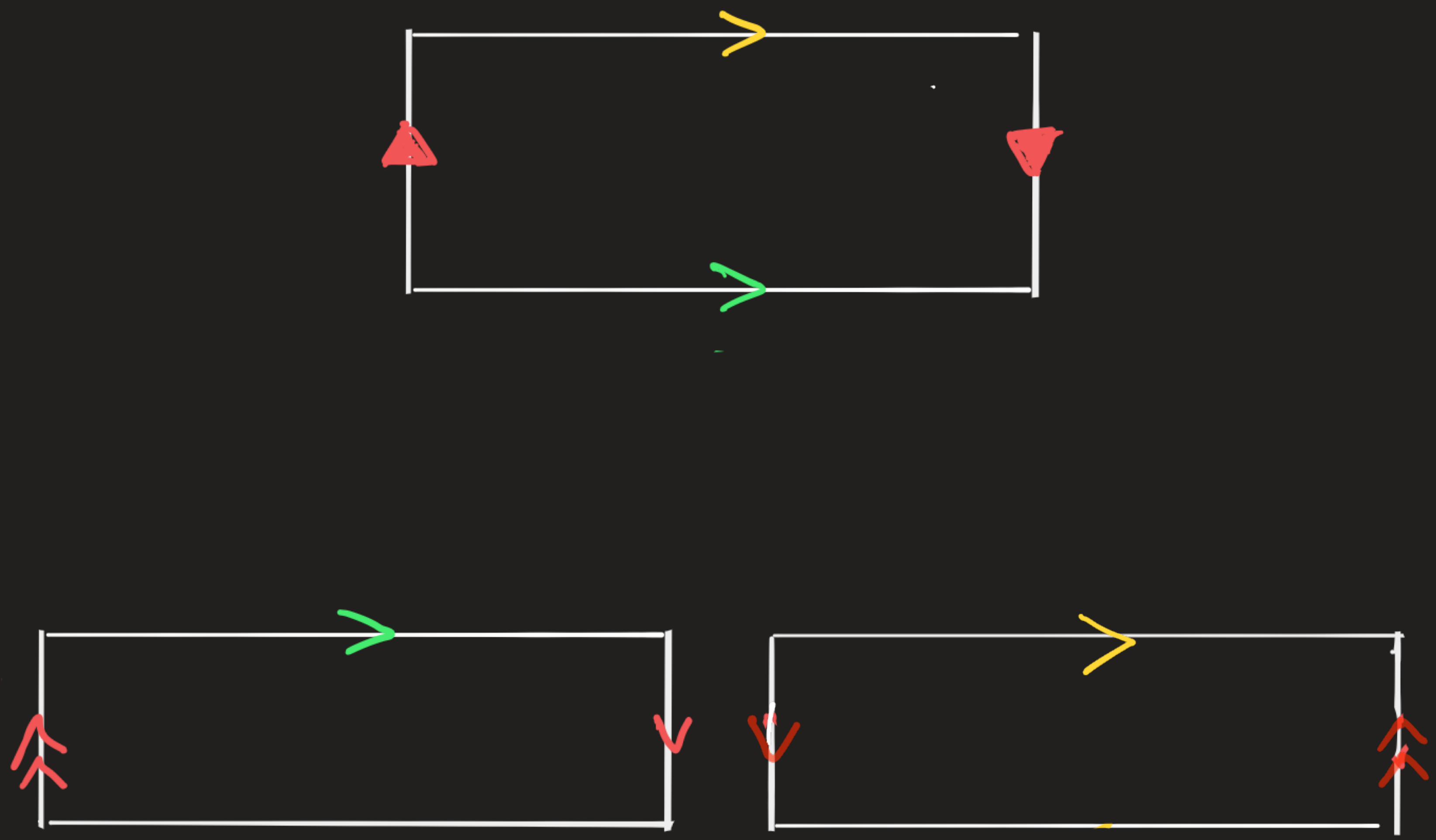
Découper la bande de Möbius



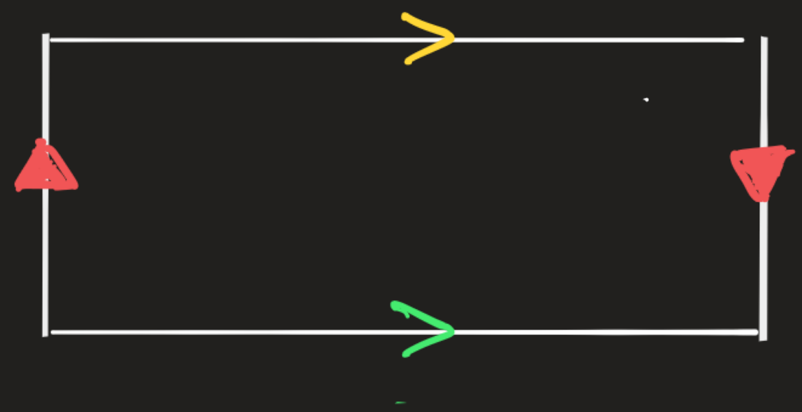
Découper la bande de Möbius



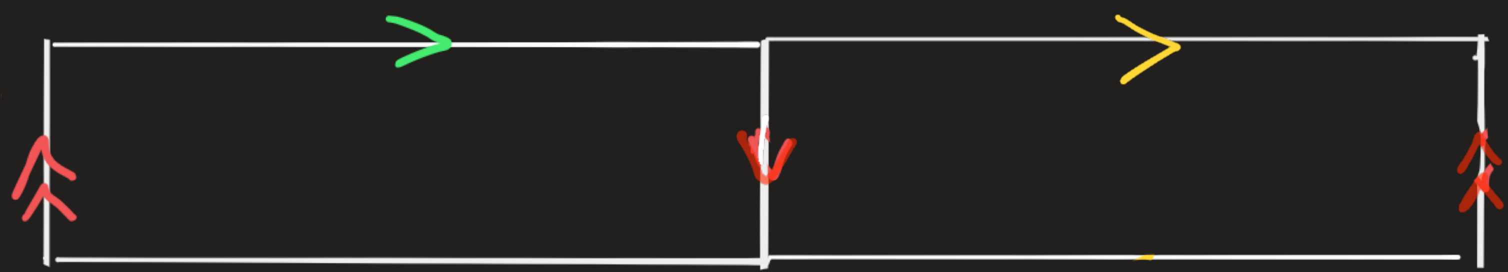
Découper la bande de Möbius



Découper la bande de Möbius



Bande de Möbius



cylindre