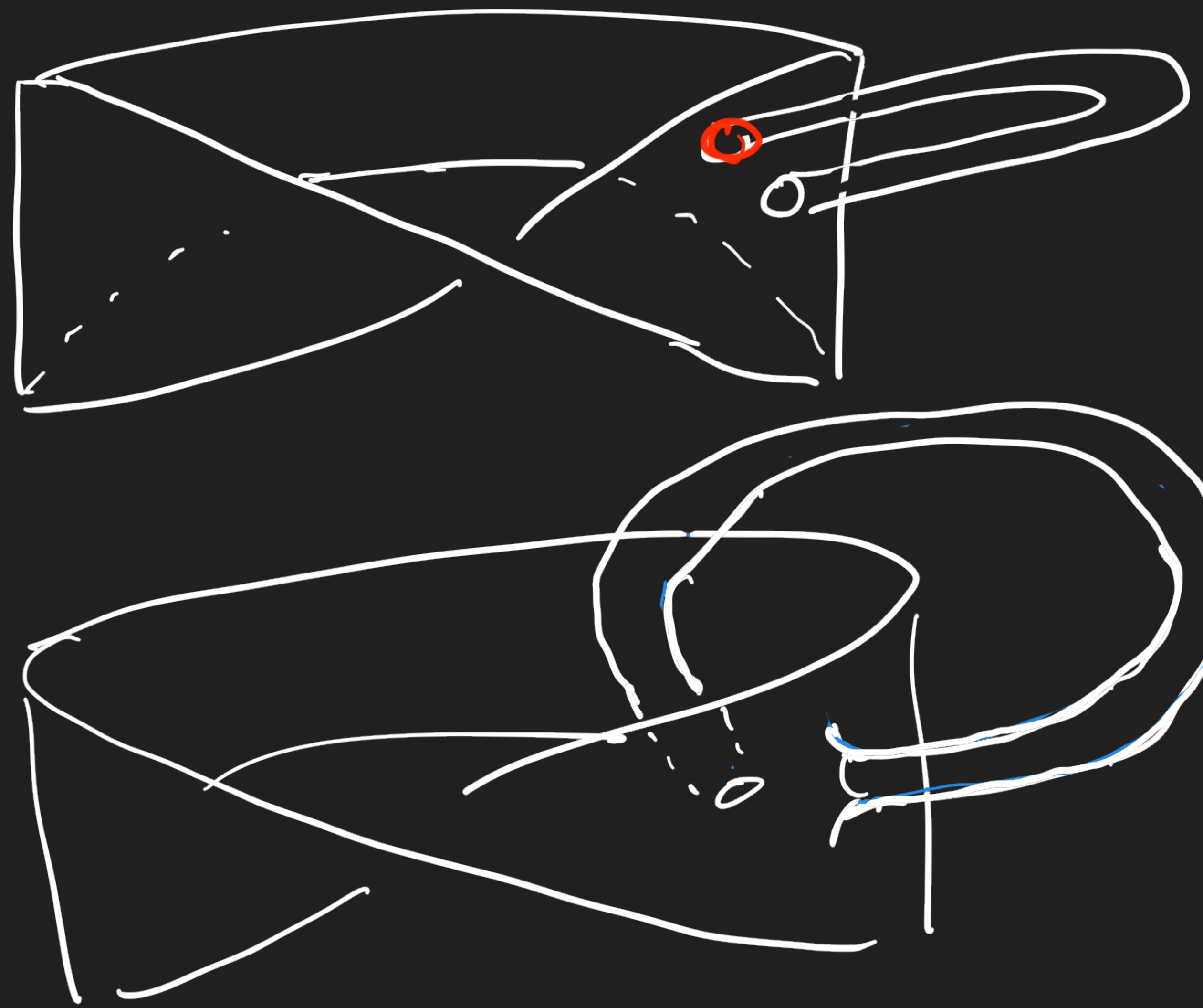
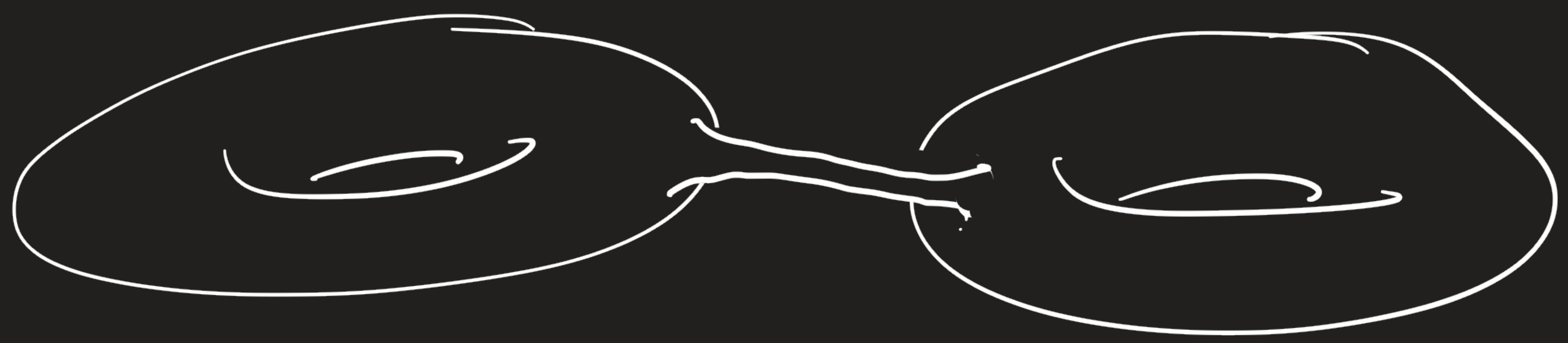
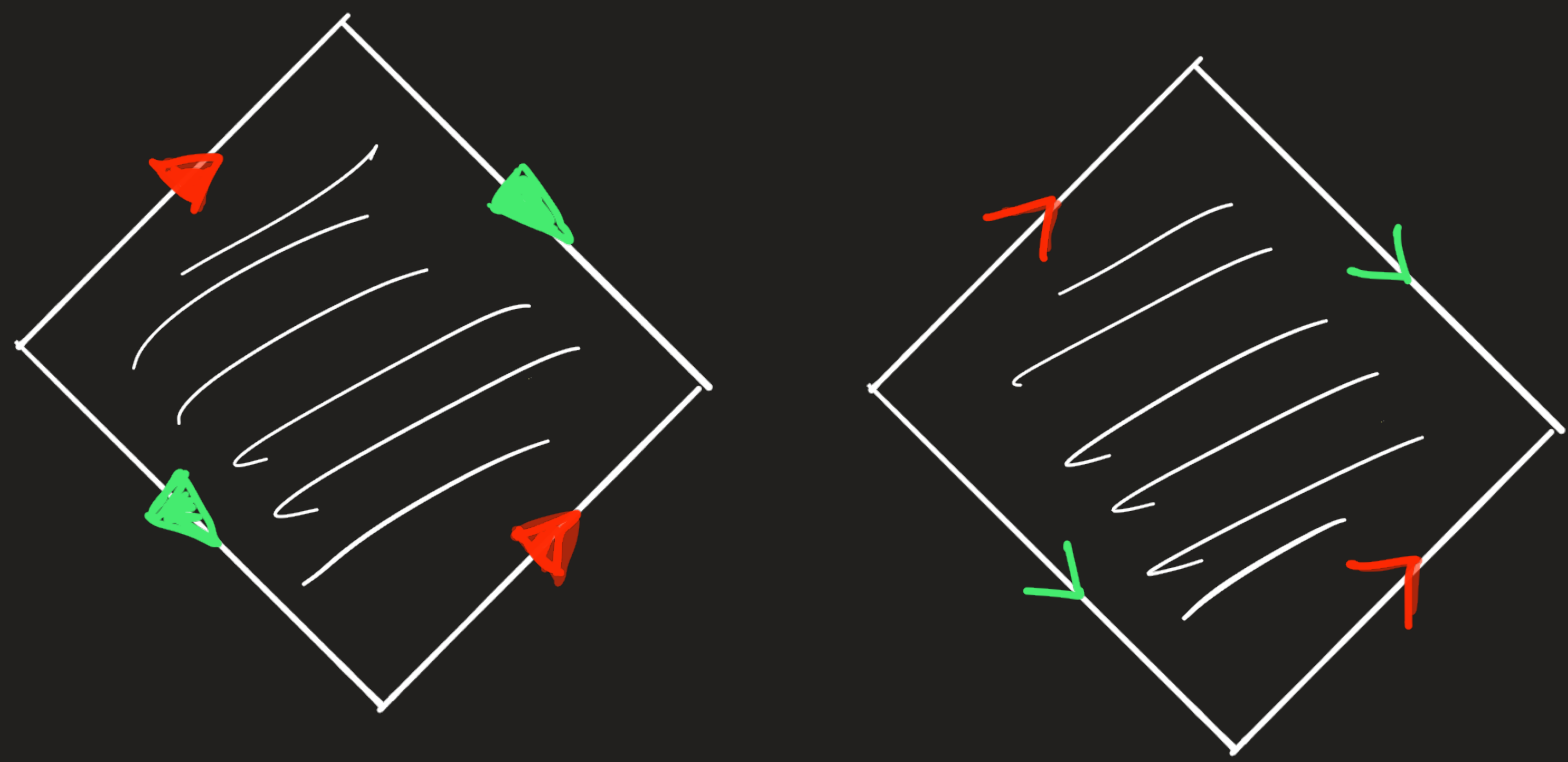




Somme connexe de deux tores.



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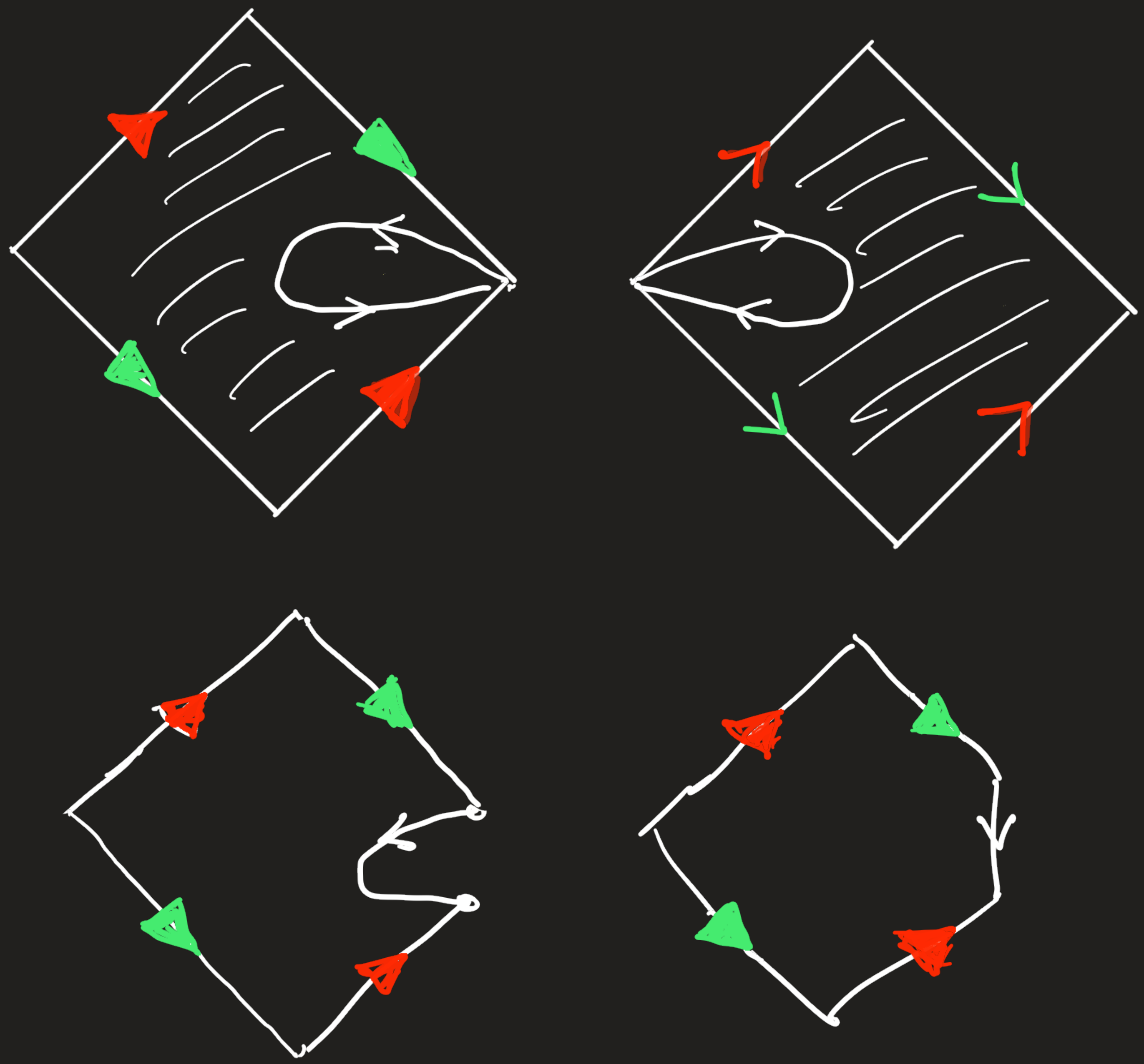
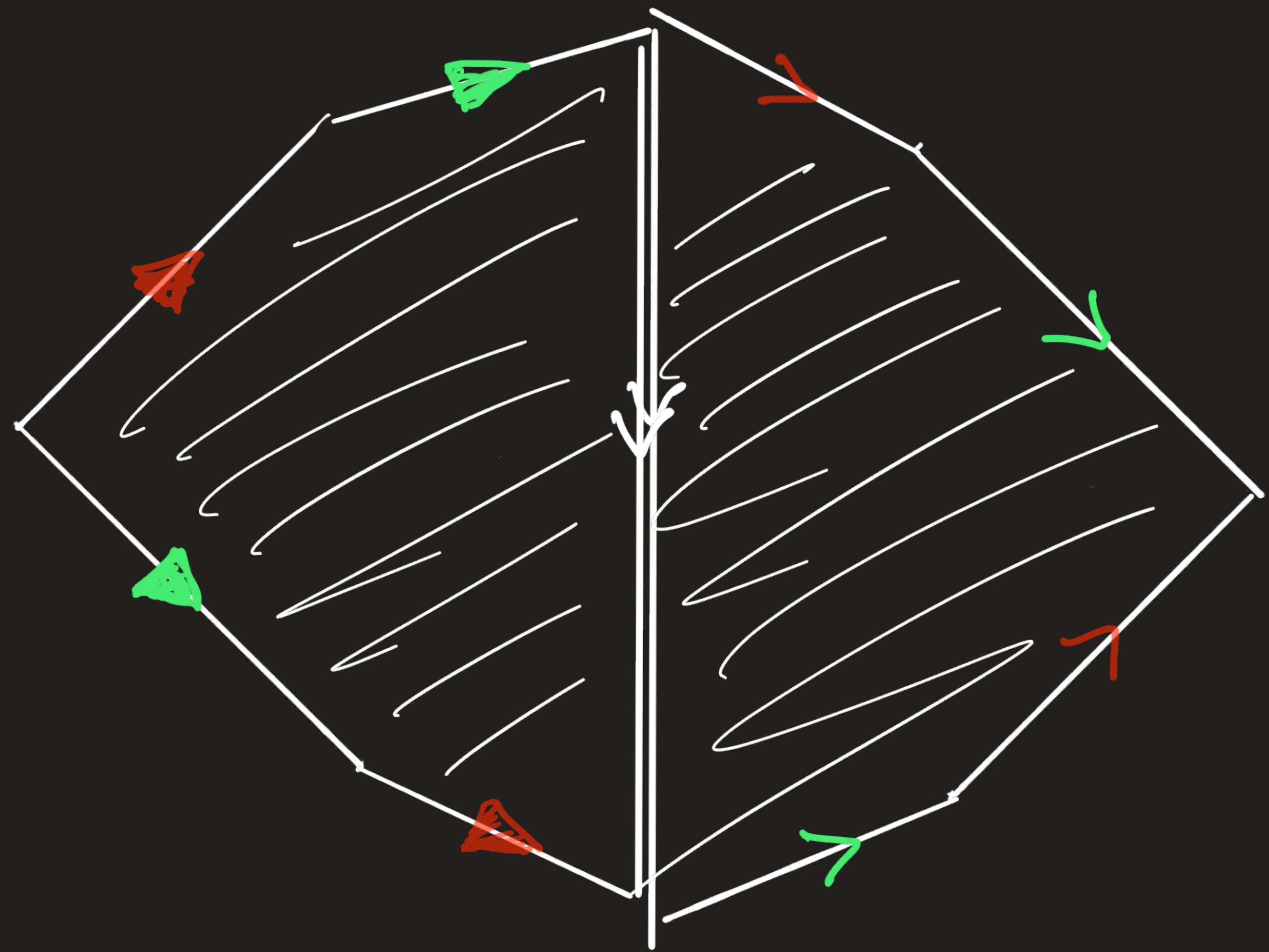
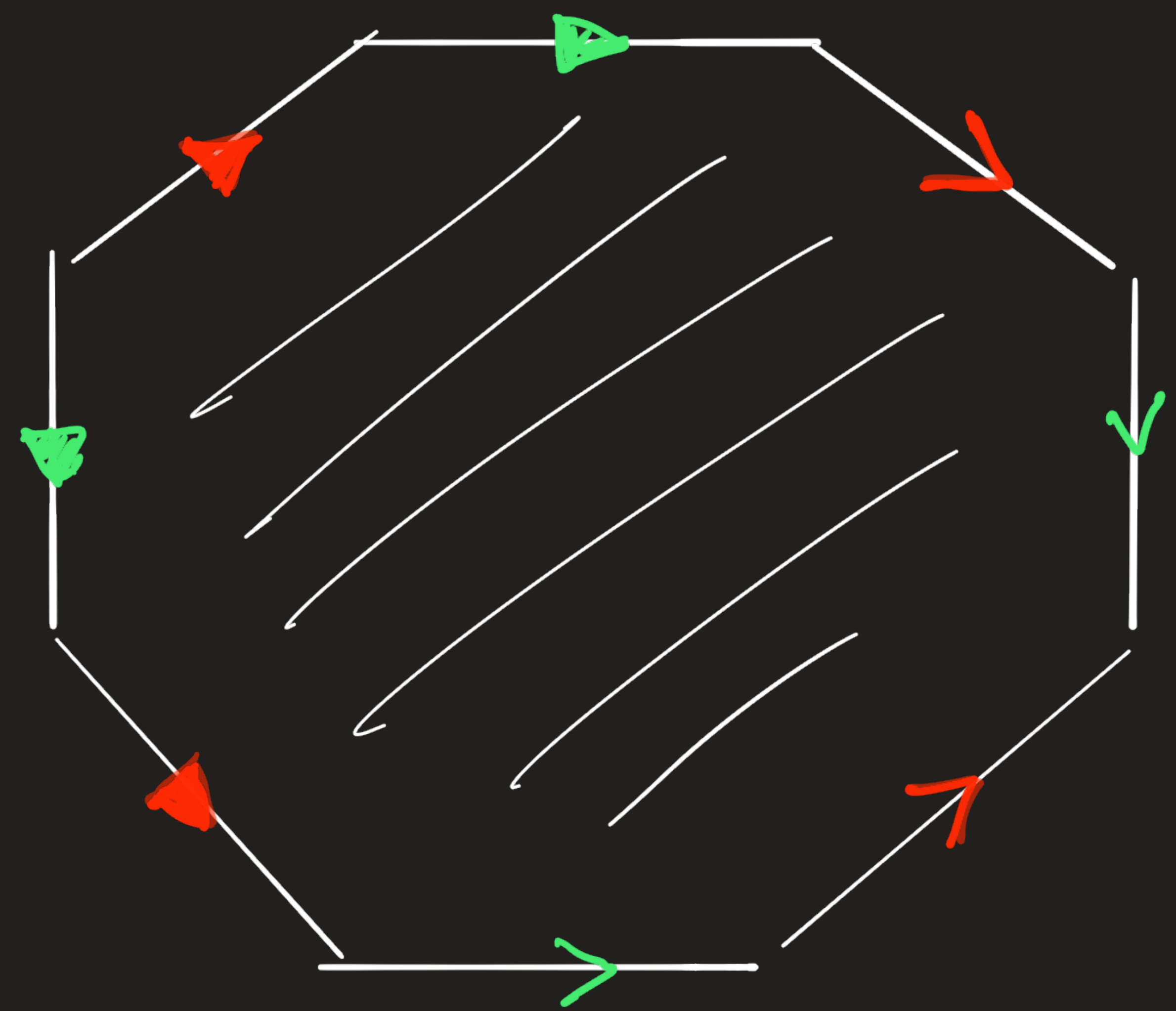


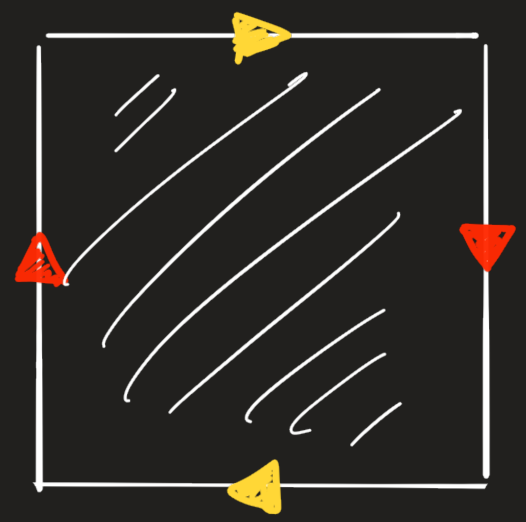
Somme connexe de deux tores.

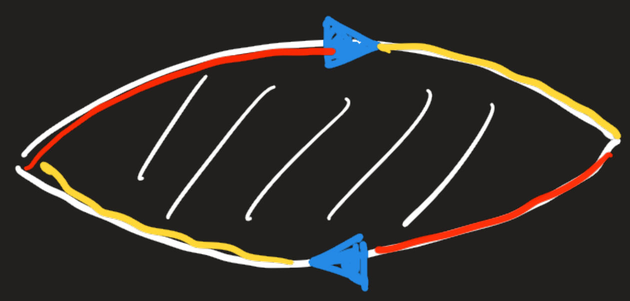


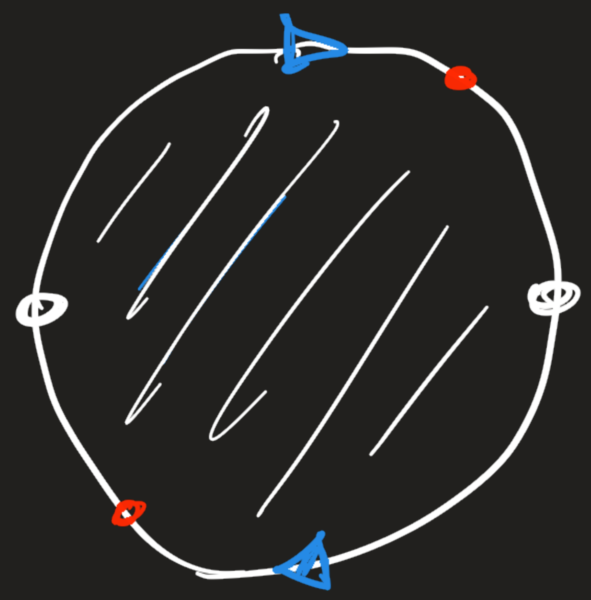
Somme connexe de deux tores.

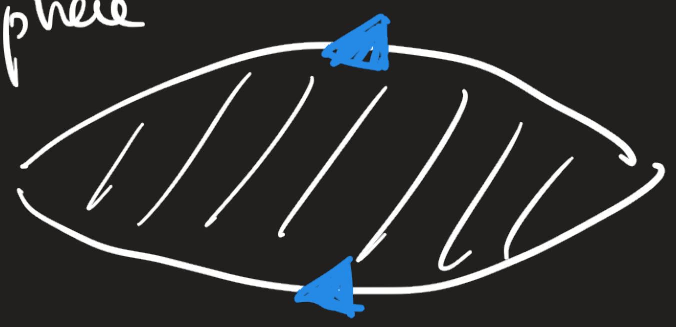


Somme connexe de deux plans projectifs

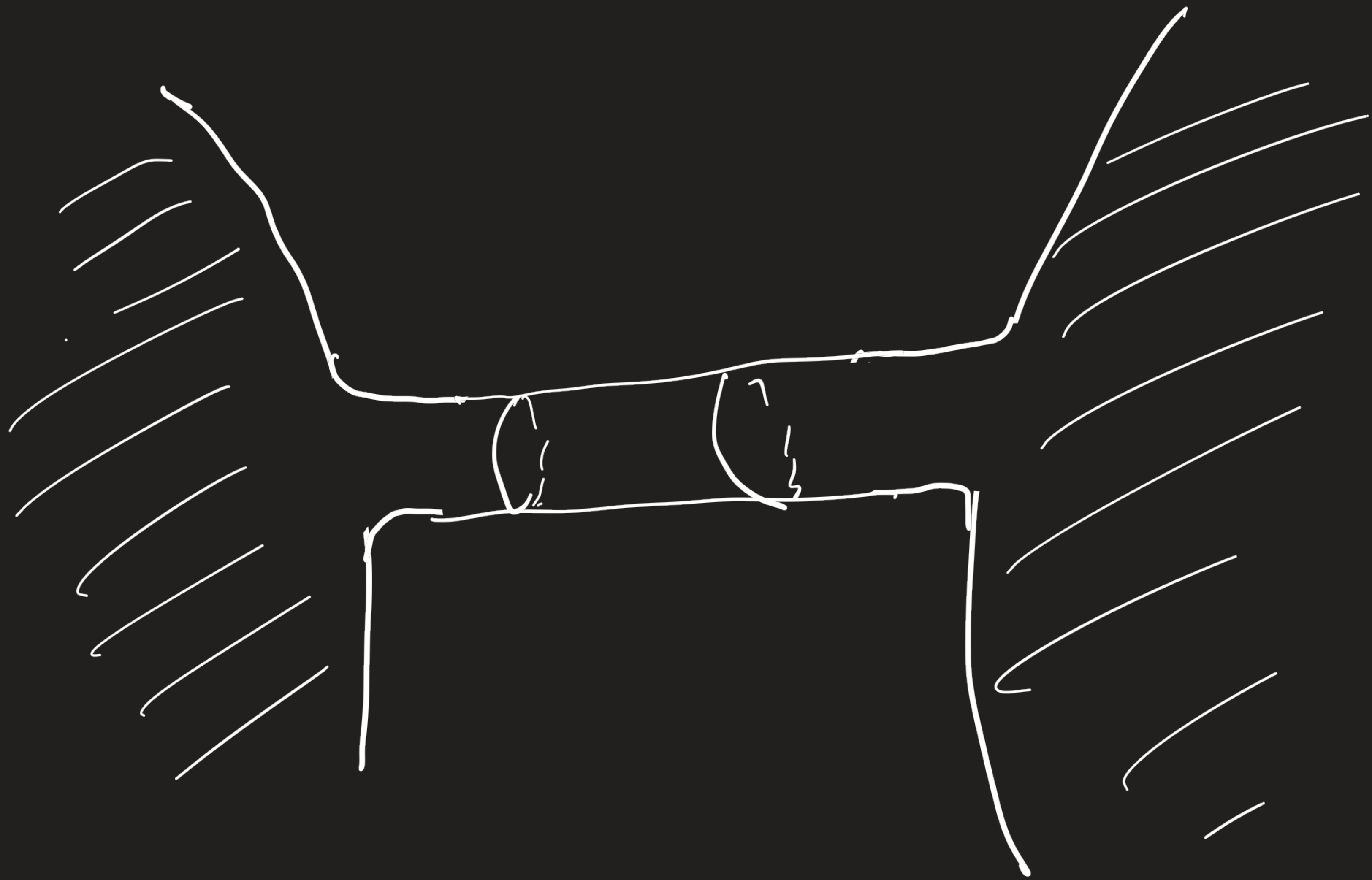




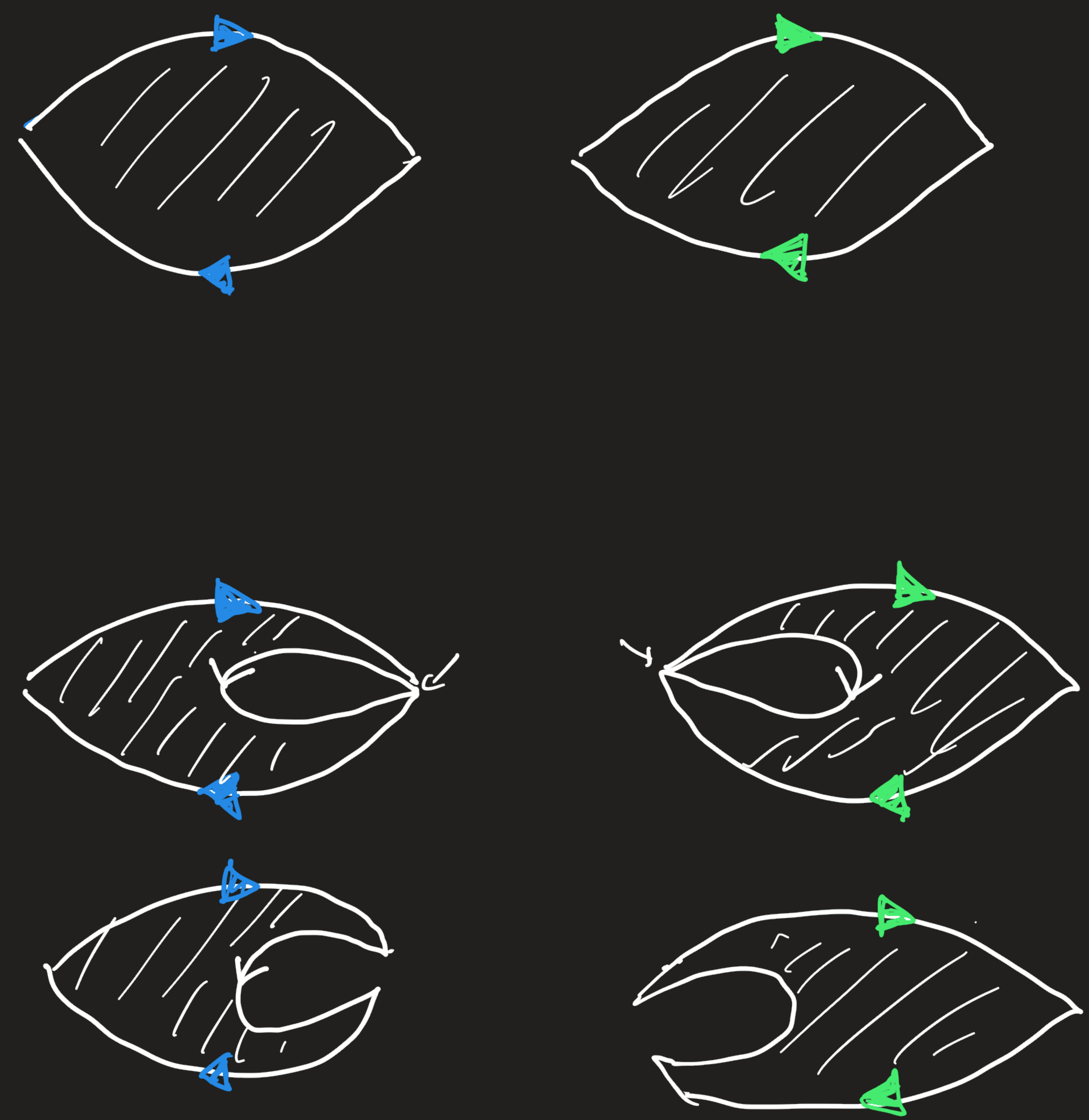




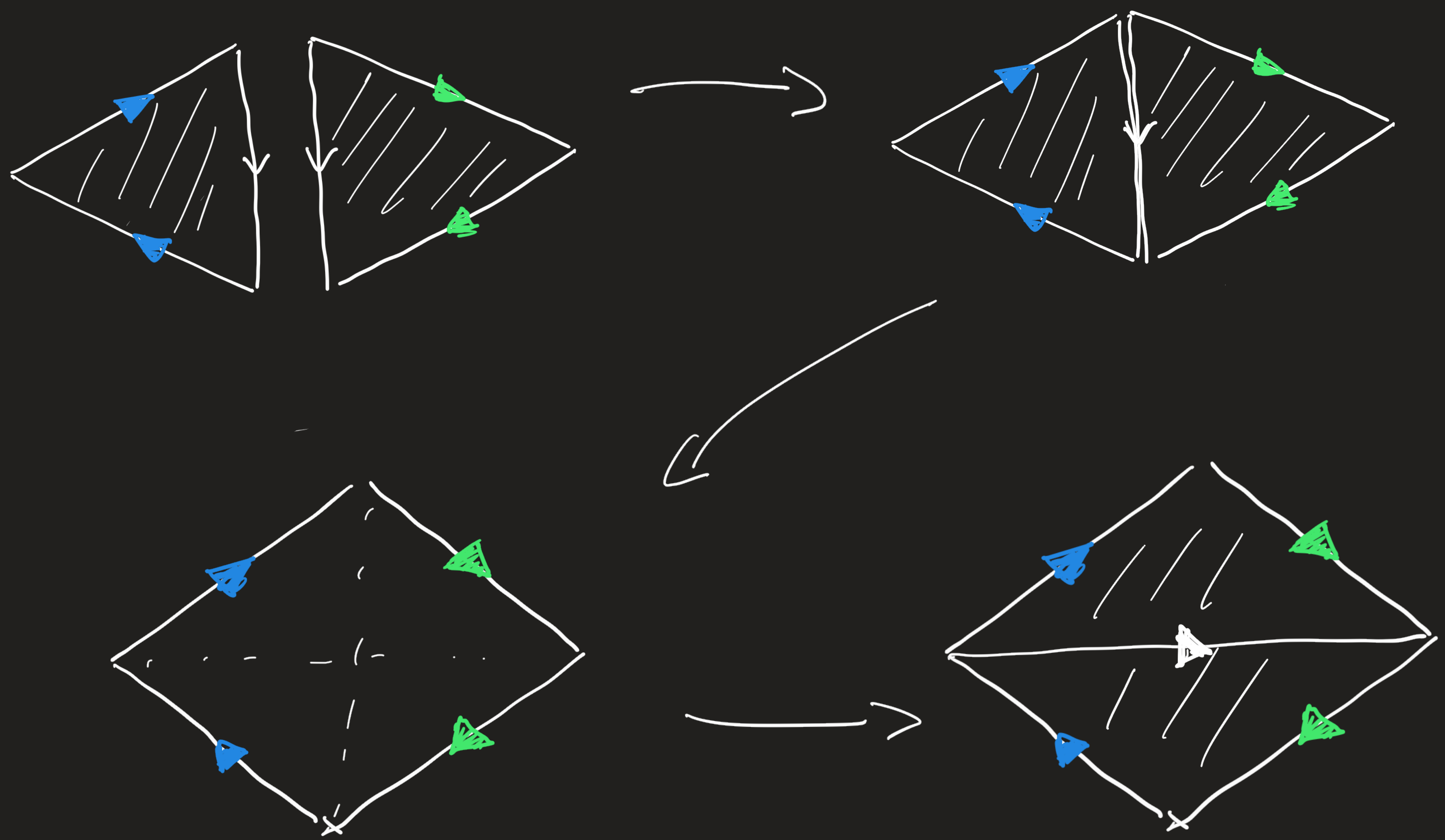
Somme connexe



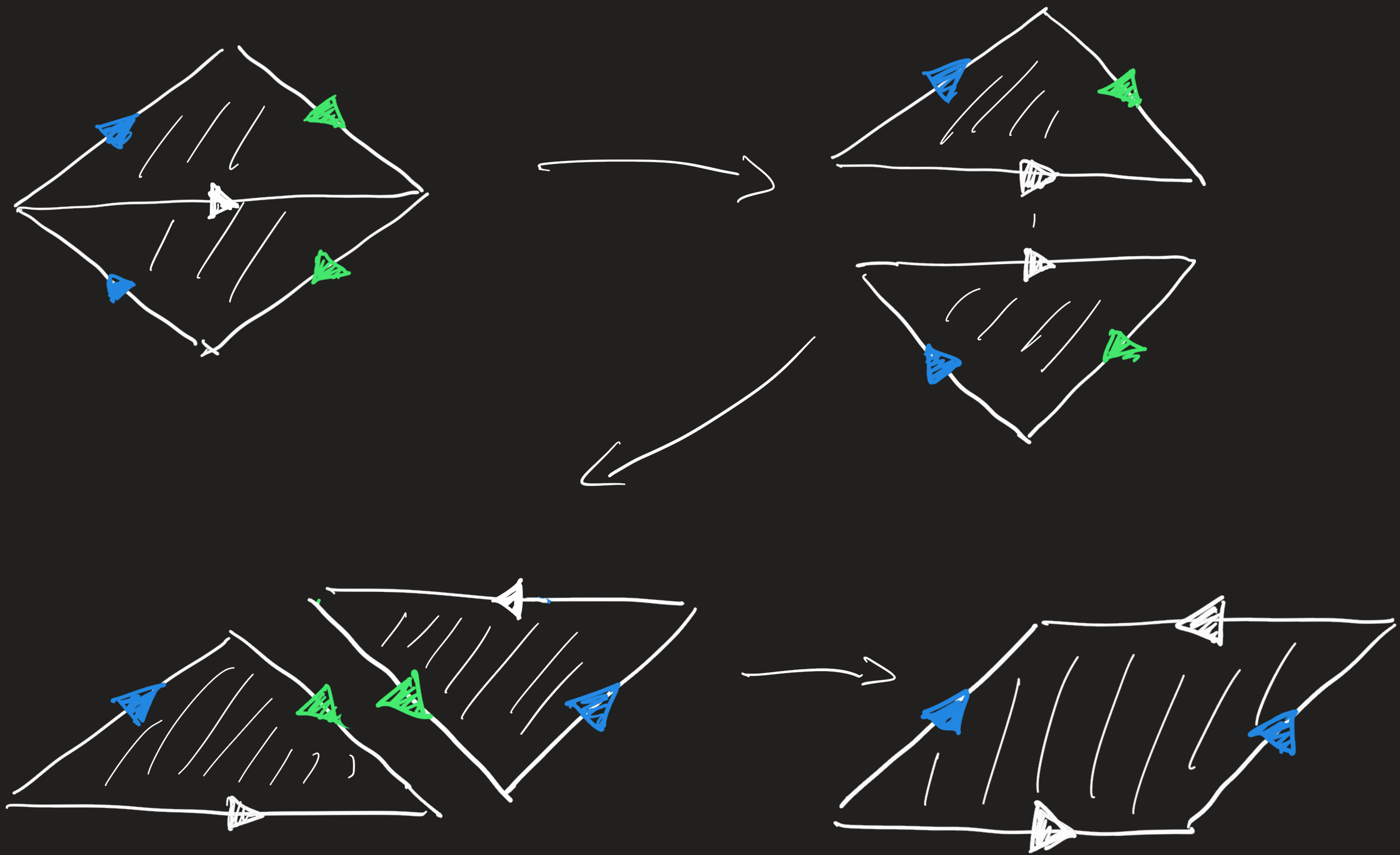
Somme connexe de deux plans projectifs



Somme connexe de deux plans projectifs

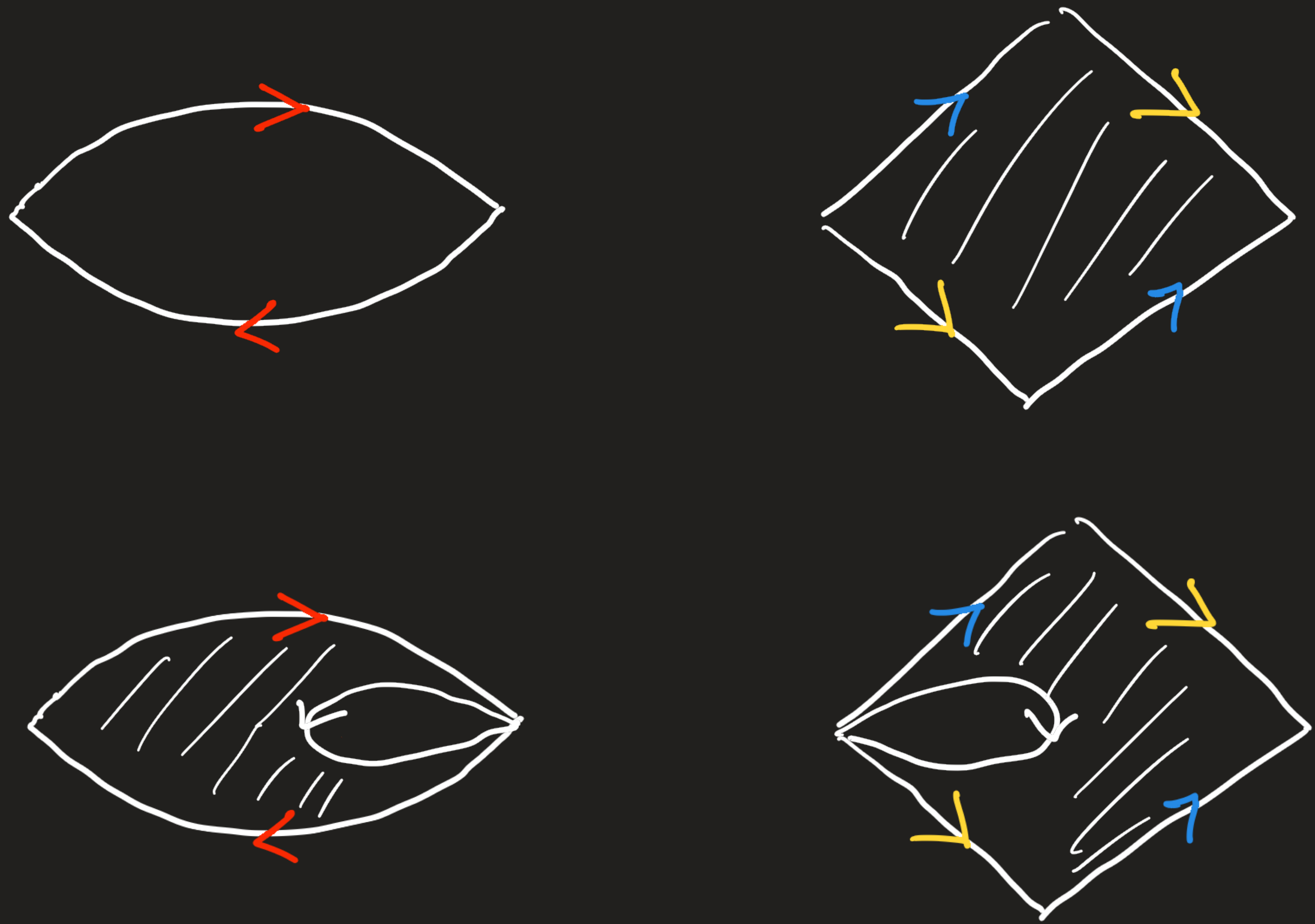


Somme connexe de deux plans projectifs



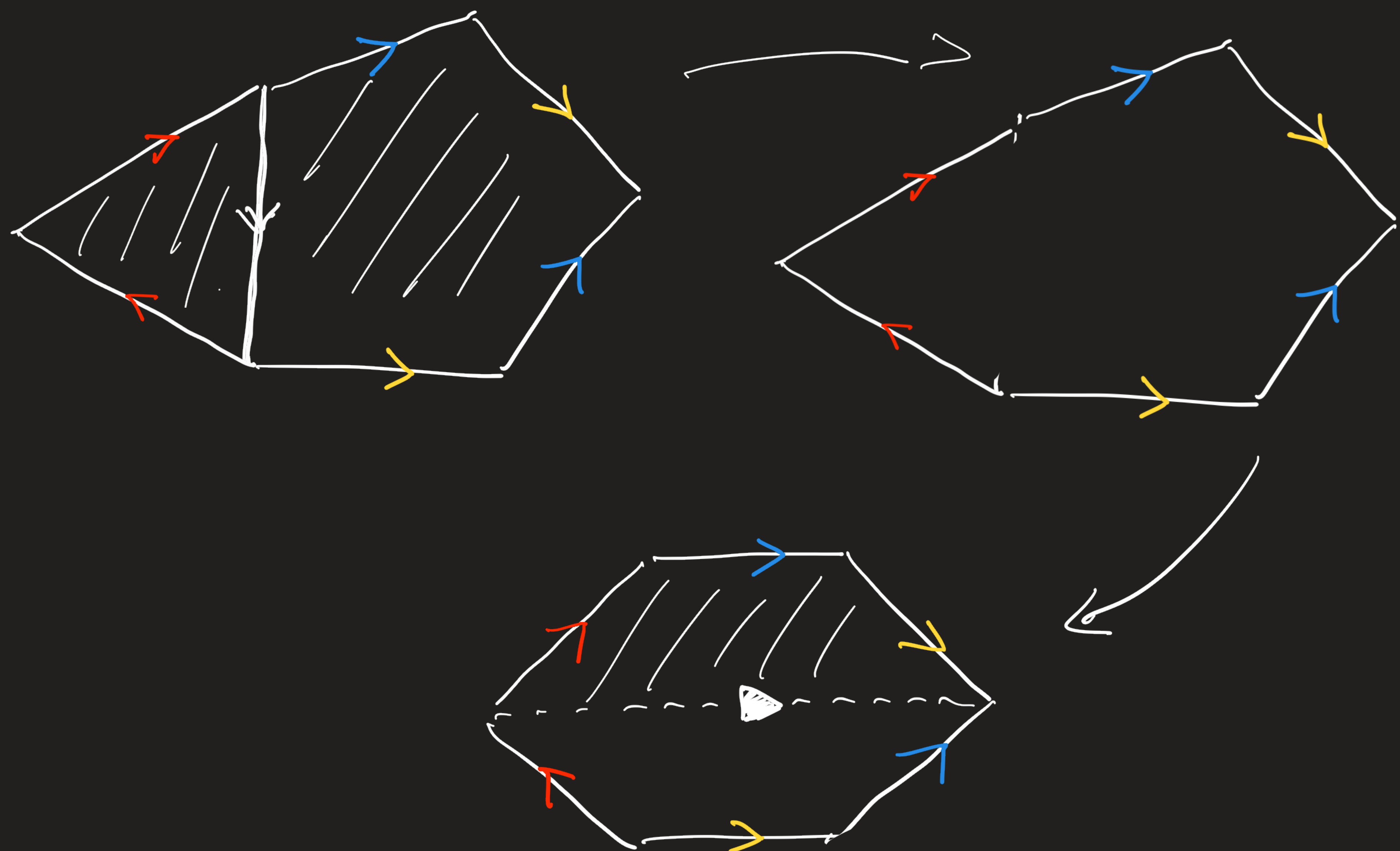
Somme connexe d'un tore et

d'un plan projectif.

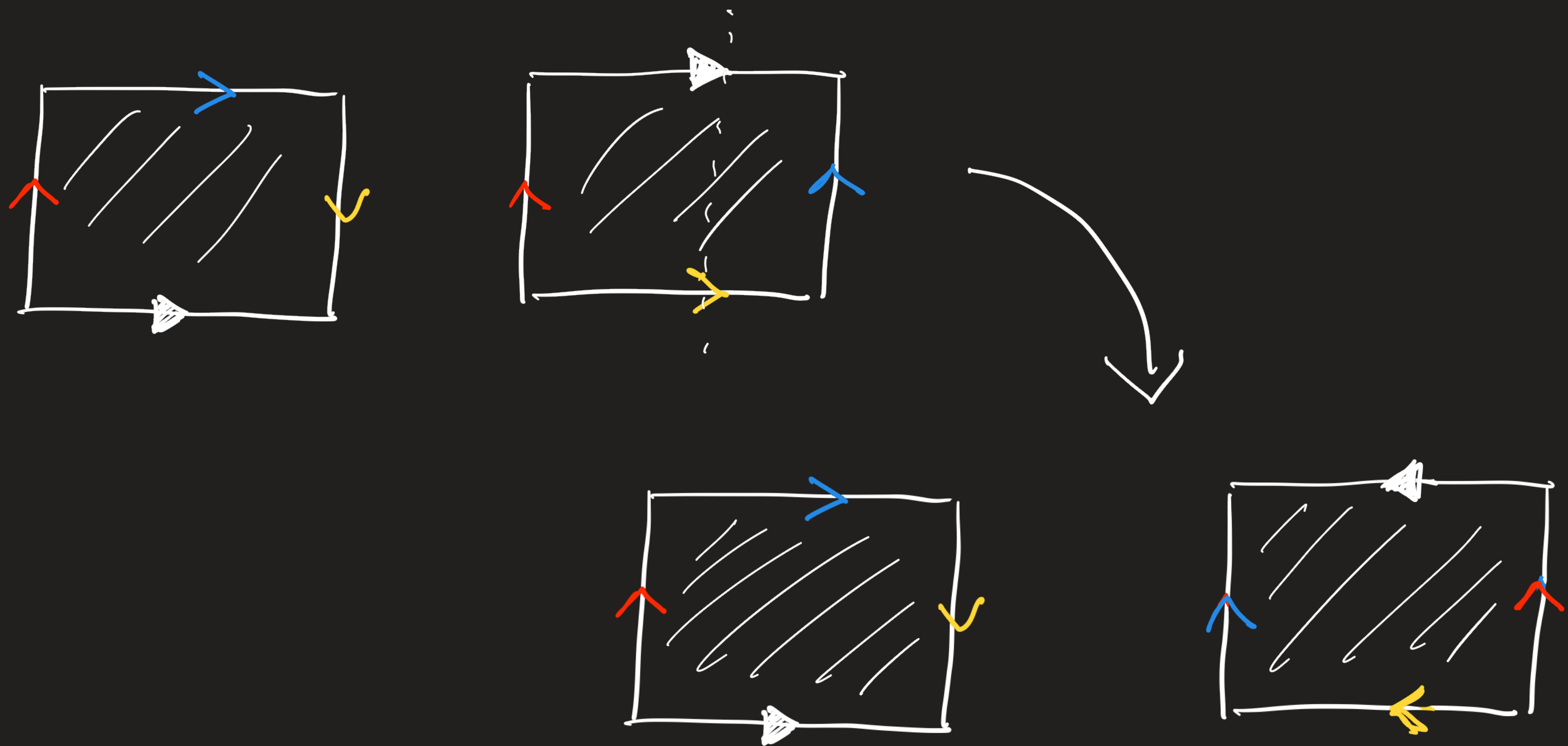


Somme connexe d'un tore et

d'un plan projectif.

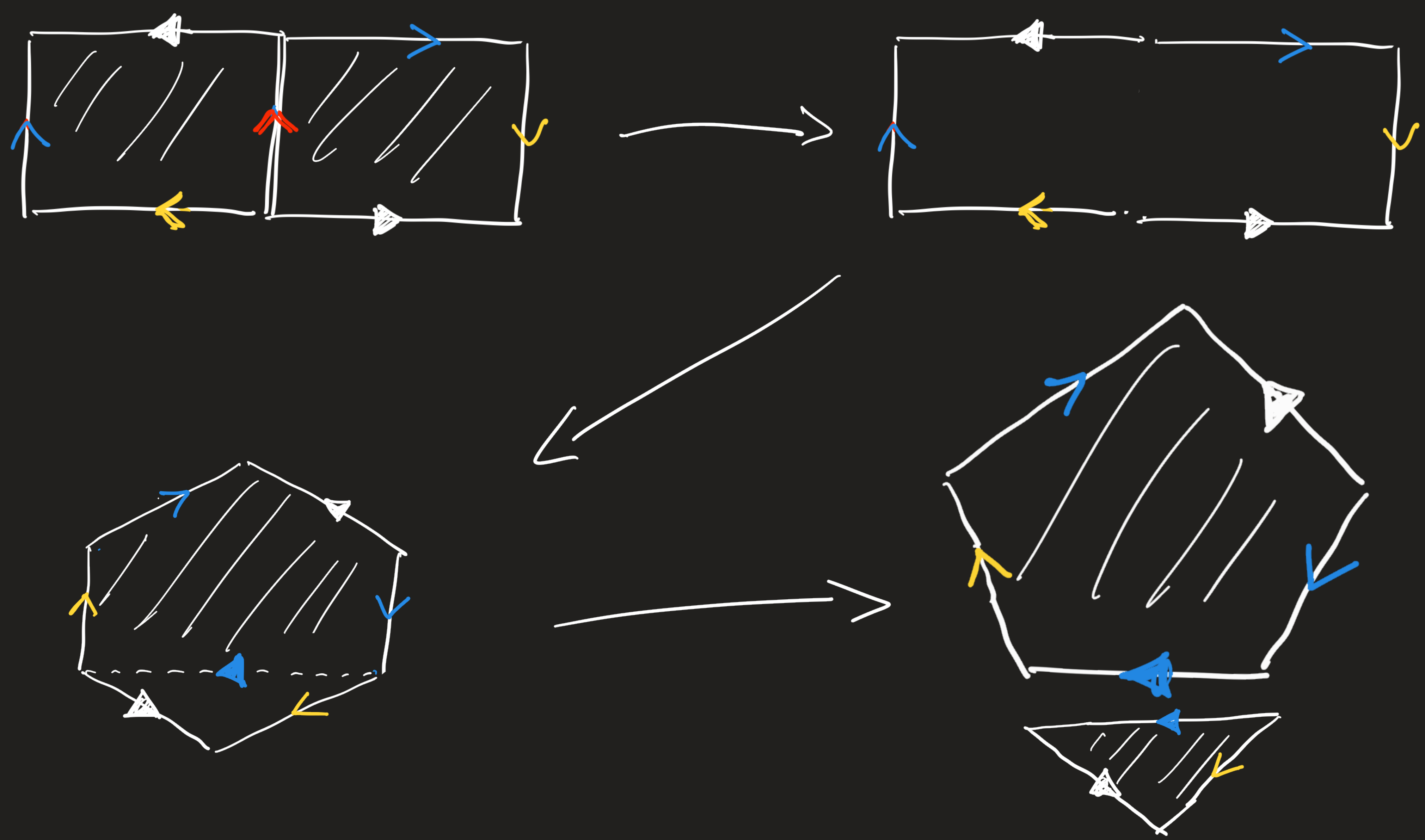


Somme connexe d'un tore et d'un plan projectif.

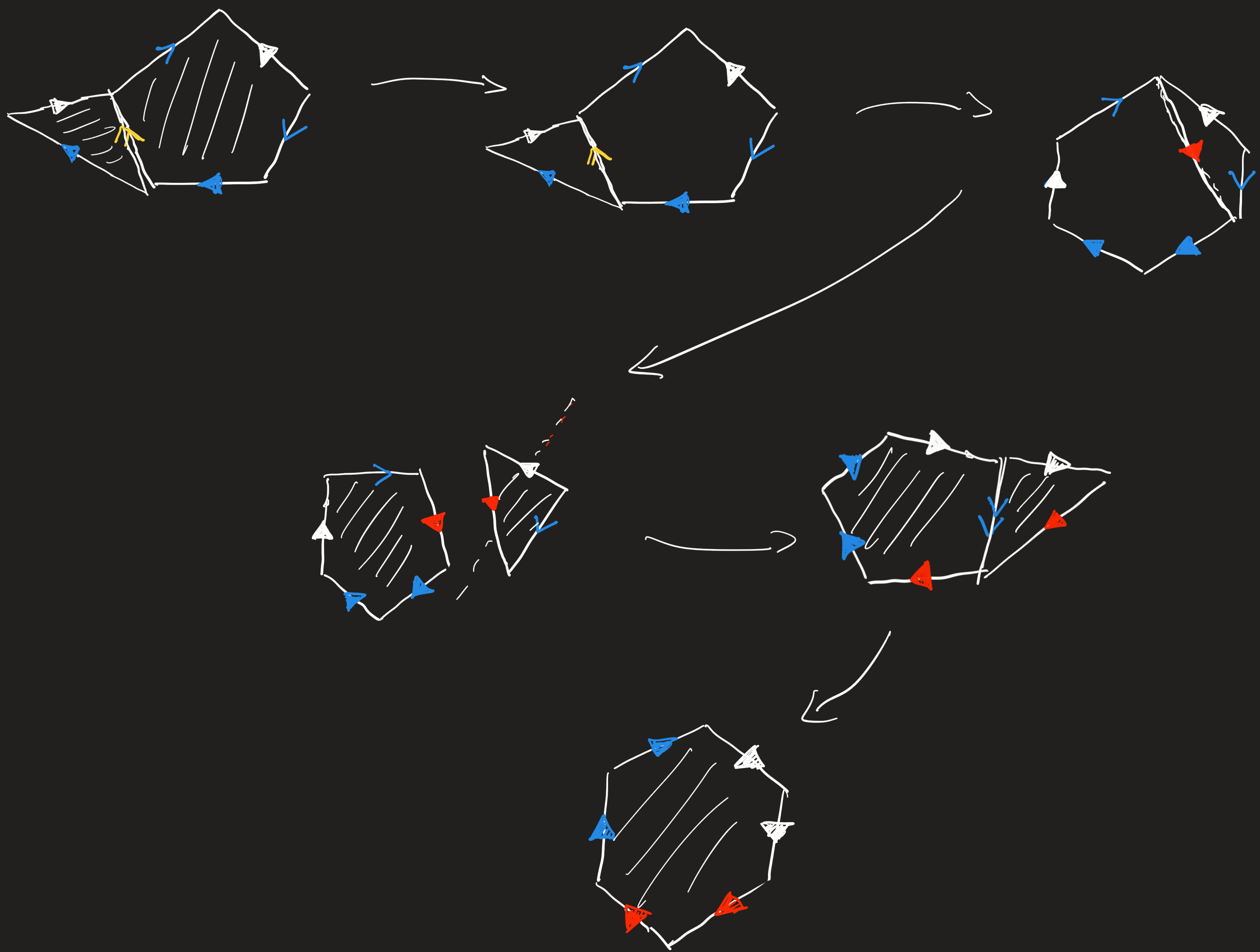


Somme connexe d'un tore et

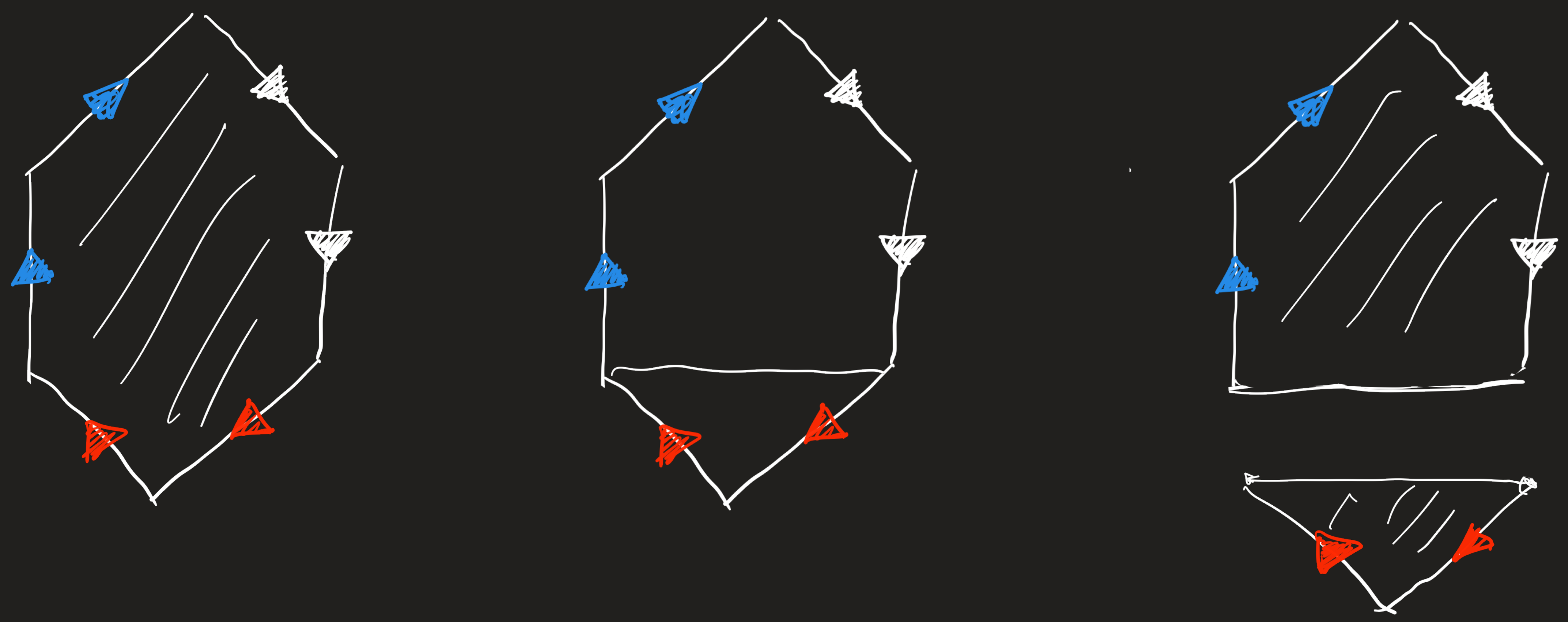
d'un plan projectif.



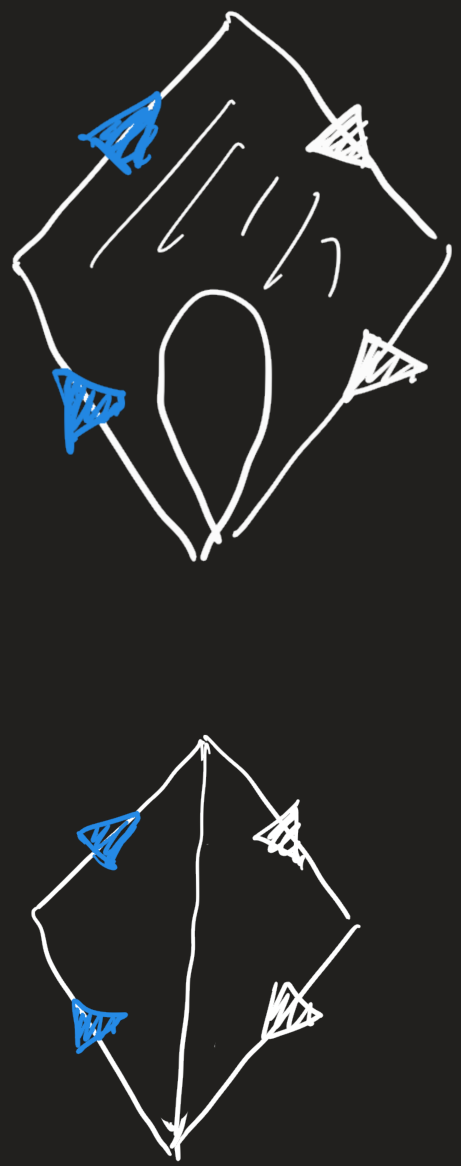
Somme connexe d'un tore et d'un plan projectif.

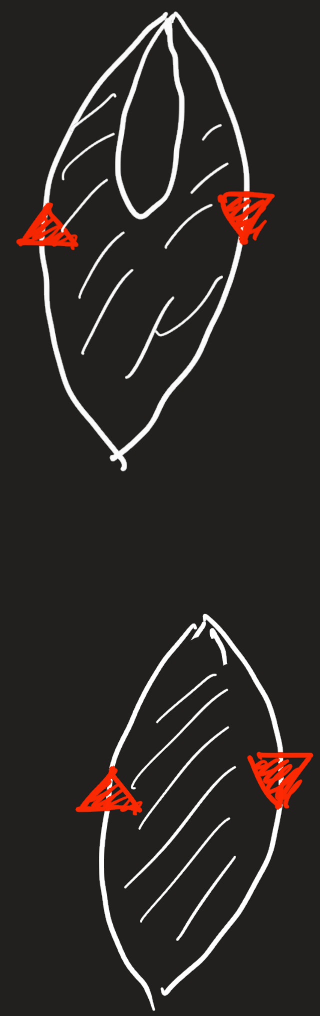


Somme connexe d'un plan projectif
et d'un tore



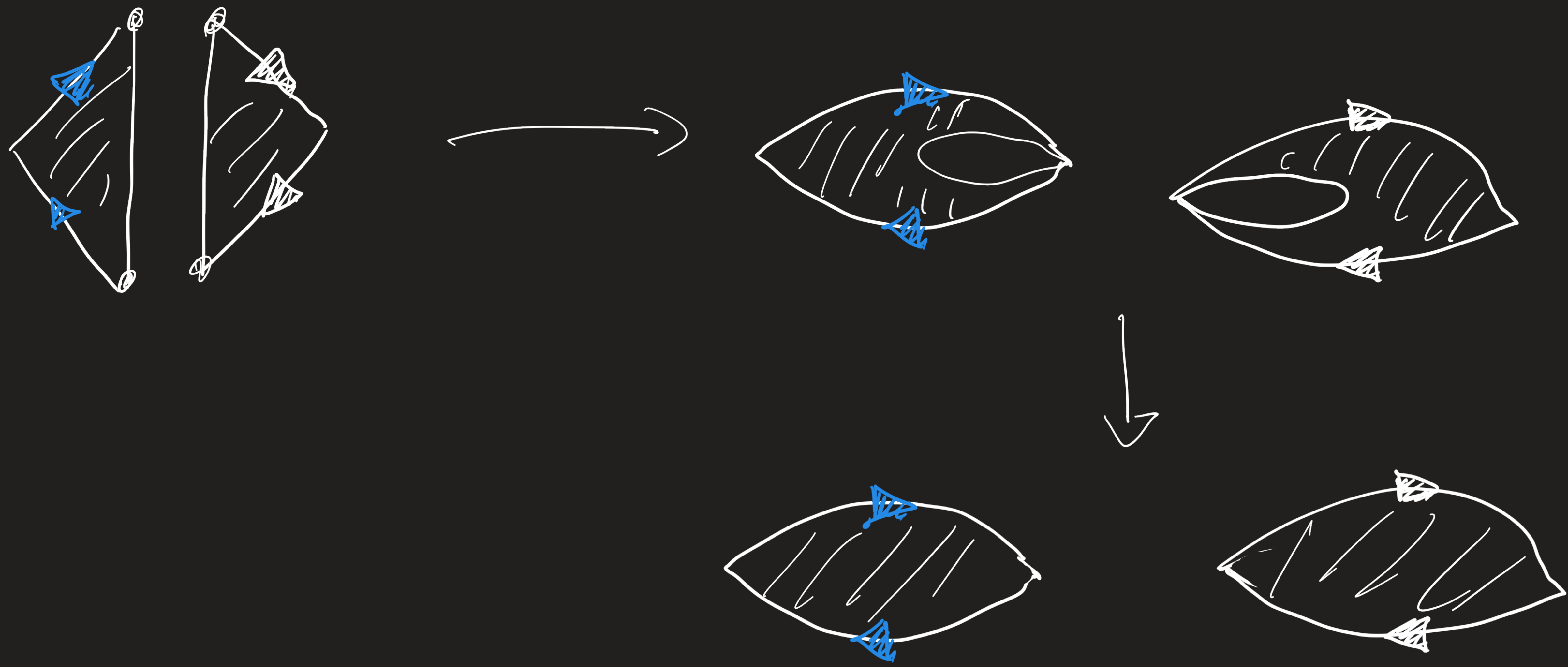
Somme connexe d'un plan projectif
et d'un tore





Somme connexe d'un plan projectif

et d'un tore



Théorème Toute surface compacte sans bord orientable est soit :

1 sphère

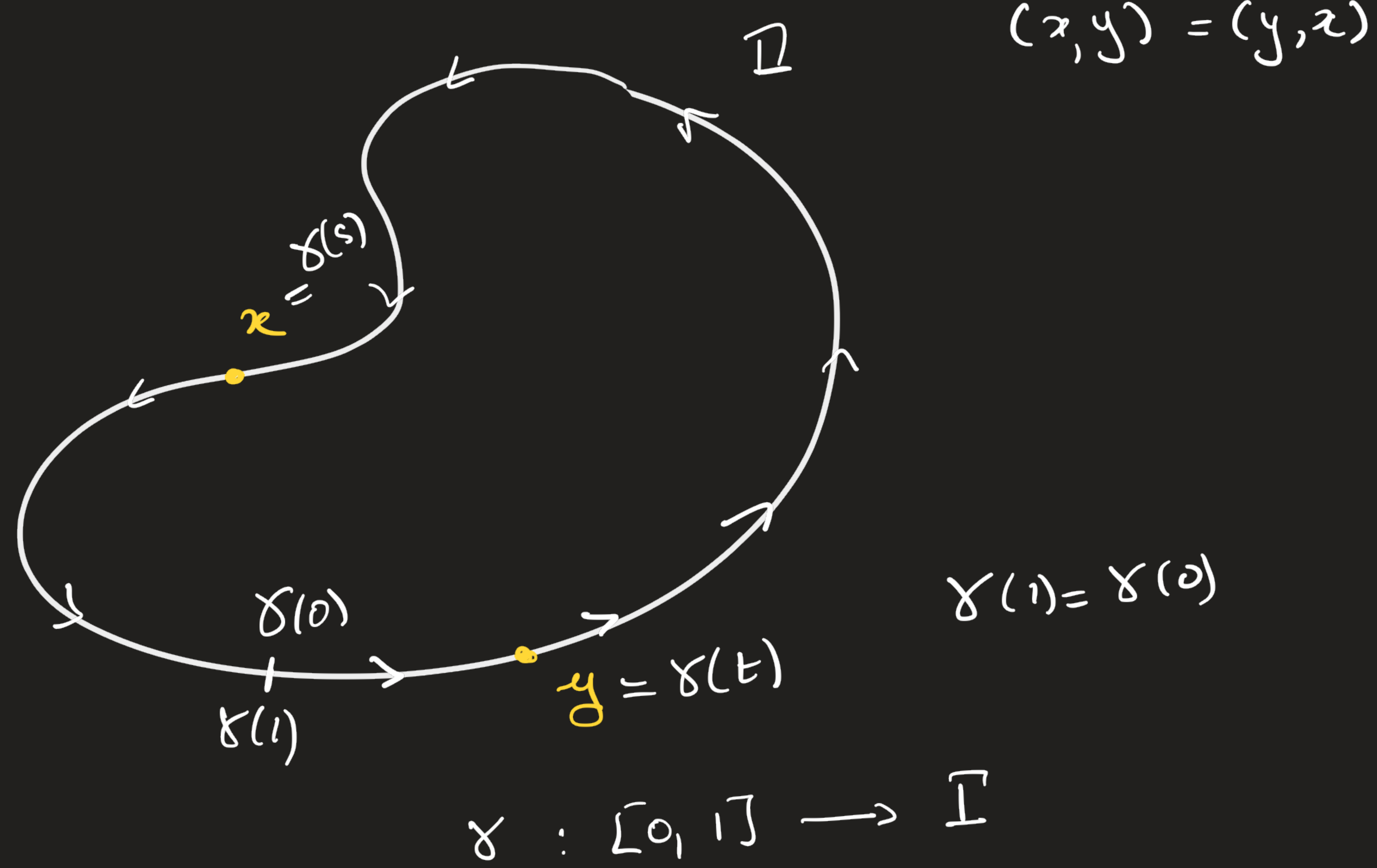
1 tore

la somme connexe de n tores

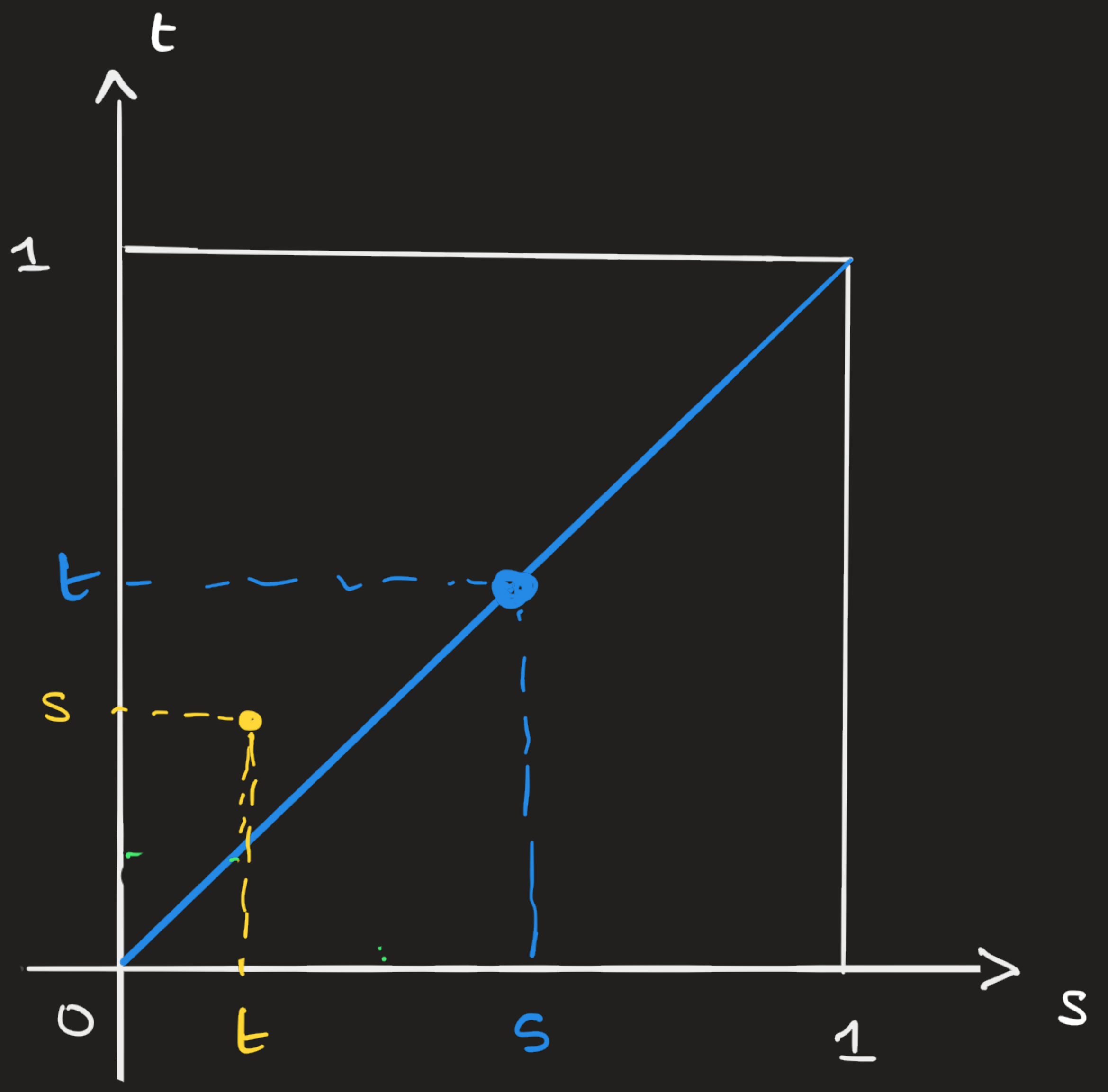
Théorème toute surface compacte sans bord non orientable est la somme connexe de n plans projectifs

Démonstration de l'existence d'un rectangle inscrit.

Preuve de P. Vaughan (1977).



$\gamma : [0,1] \longrightarrow \Gamma$ $\qquad$ $\gamma(0) = \gamma(1)$



(x, y)

$x, y \in \Gamma$

$\gamma(s), \gamma(t)$

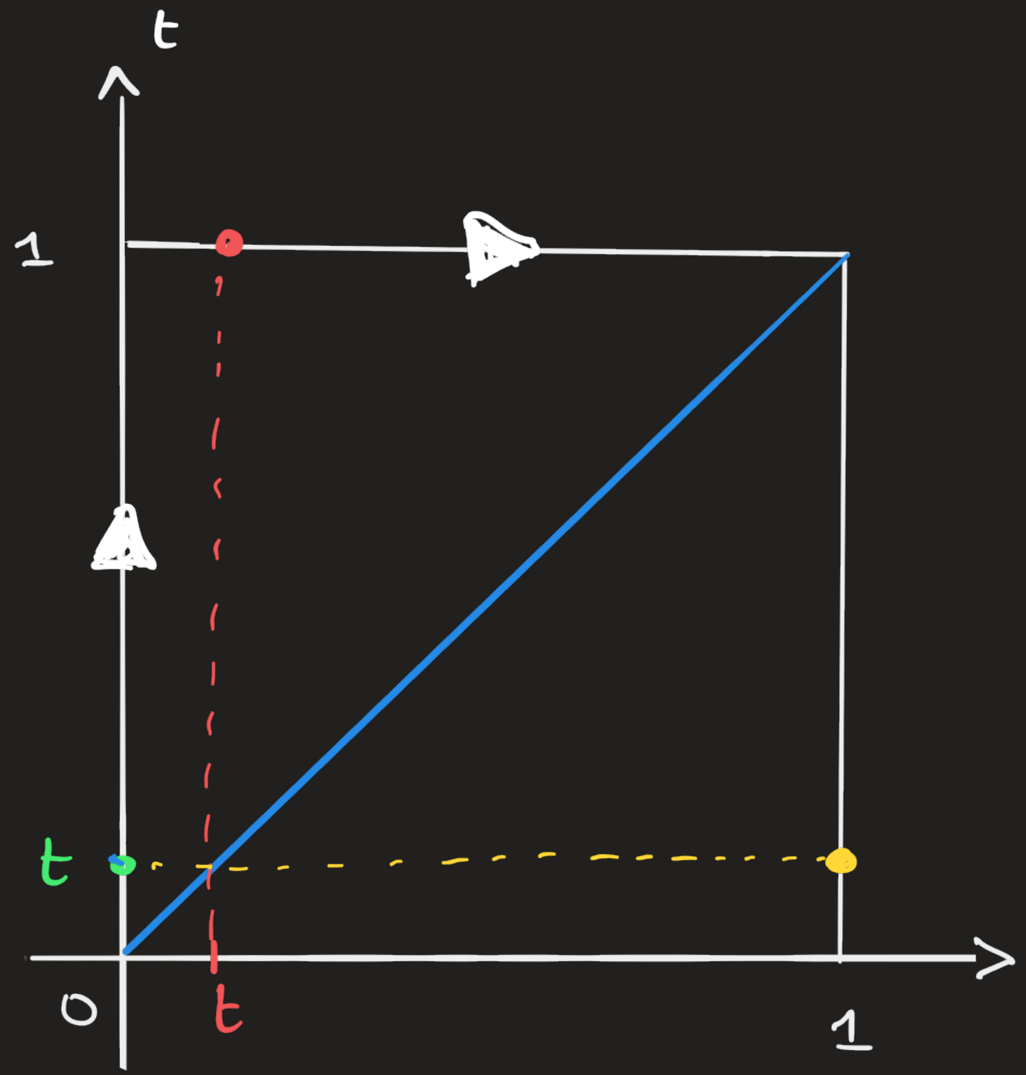
$\;\;\, \| \qquad\;\; \|$

$\;\;\, x \qquad\;\; y$

$0 \leq s \leq 1$

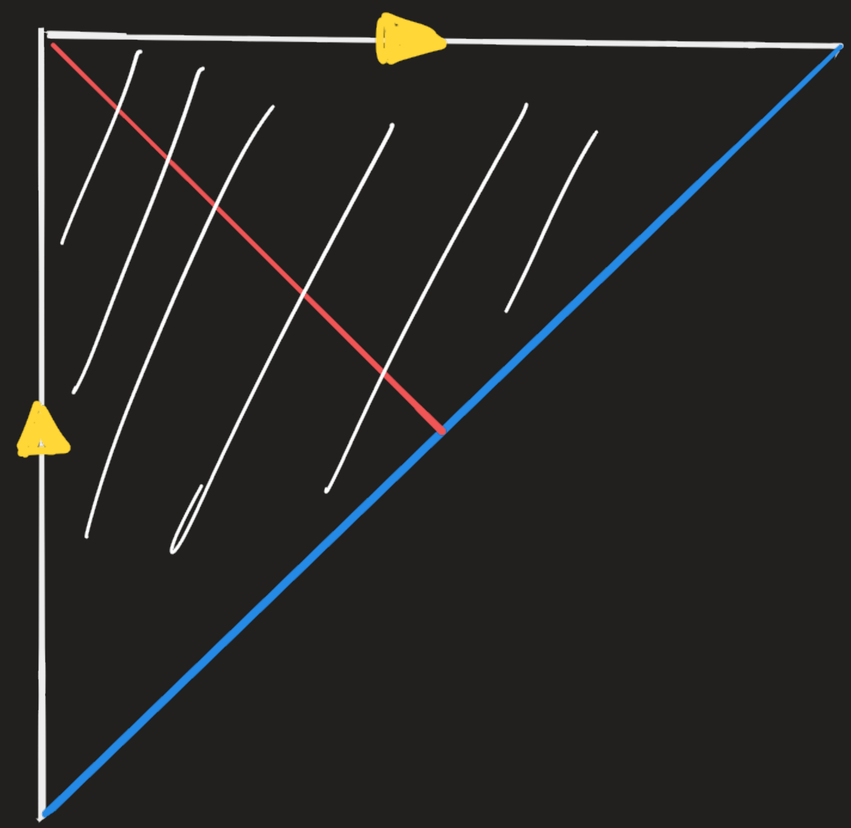
$0 \leq t \leq 1$

$\gamma : [0,1] \longrightarrow I$ $\qquad \gamma(0) = \gamma(1)$

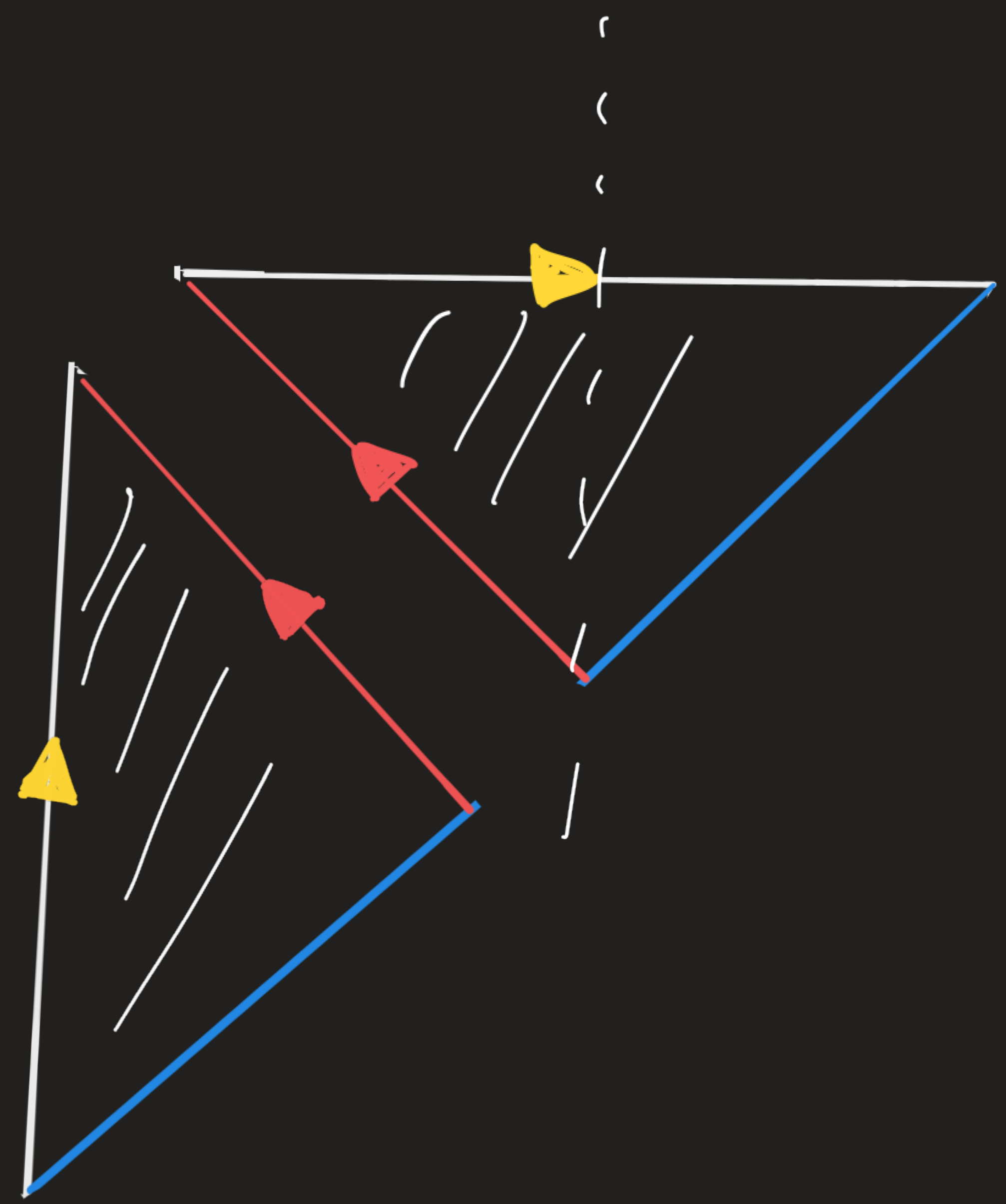


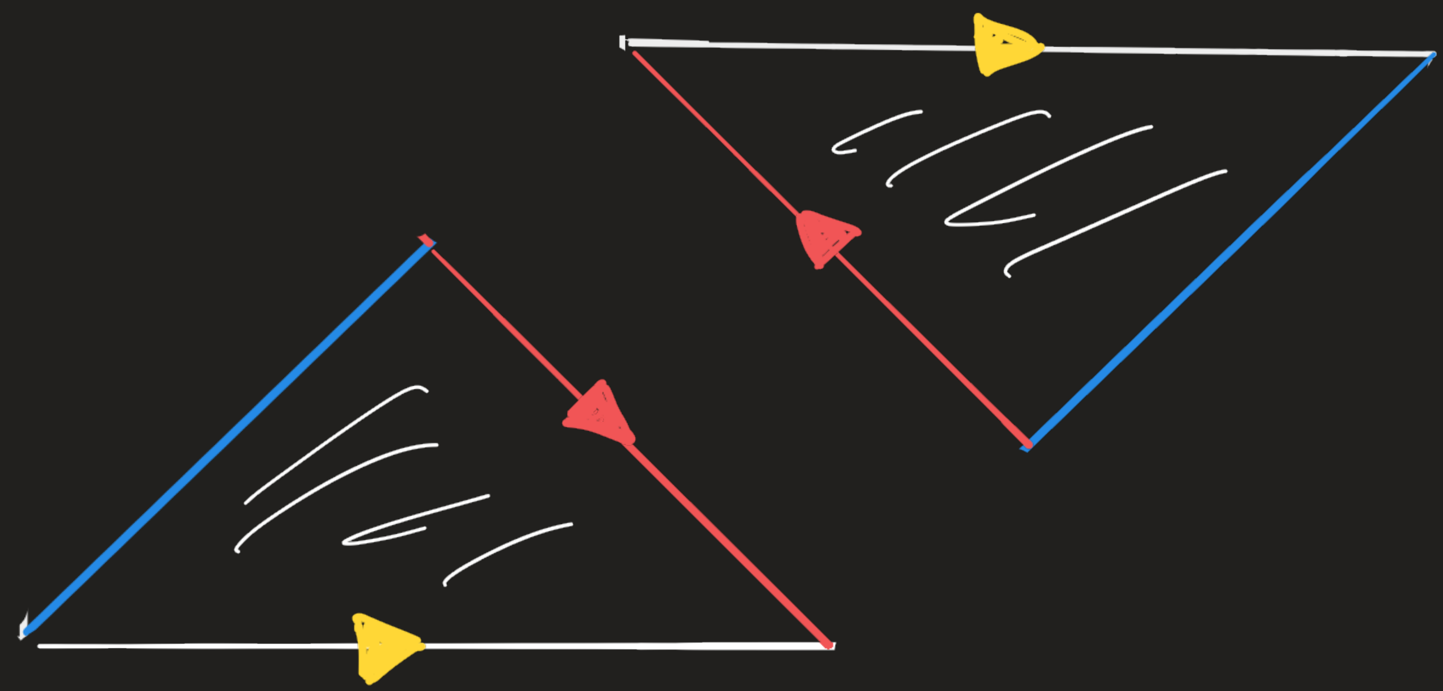
$\gamma(0)\,,\ \gamma(t)$

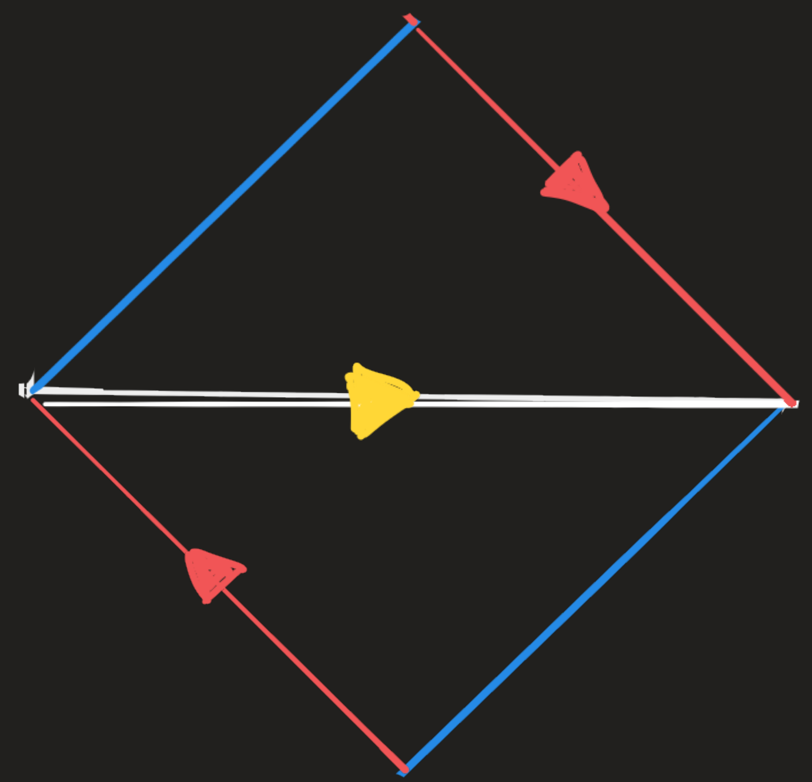
$\gamma(t)\,,\ \gamma(1) = \gamma(0)$



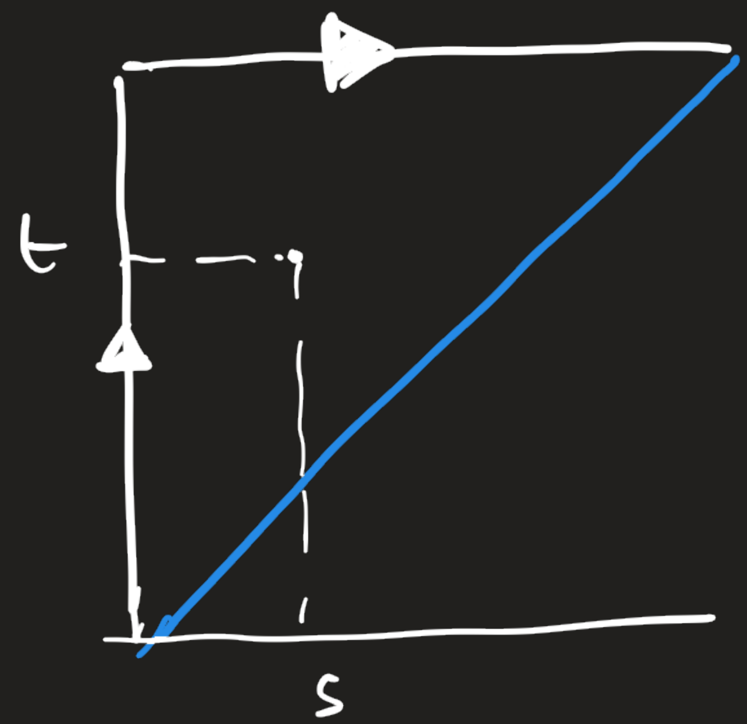
Bande de Möbius







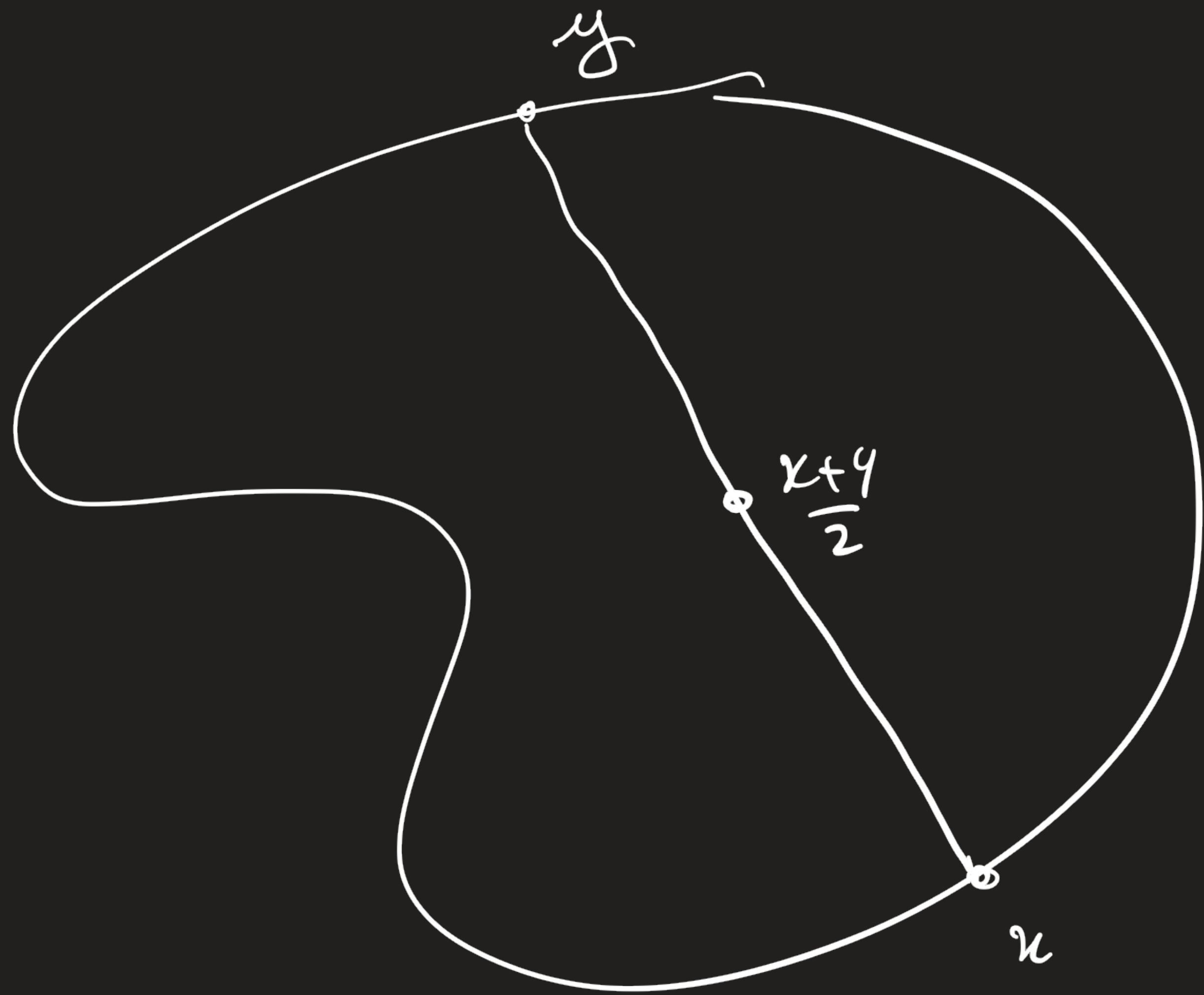
Bande de Möbius



$x = \gamma(s)$

$y = \gamma(t)$

$$\Phi(x, y) = \left(\frac{x+y}{2}, |x y| \right)$$



$$\Phi(x,y) = \left(\frac{x+y}{2}, |xy|\right)$$

$$\Big(\underbrace{\frac{x+y}{2}}_{\mathbb{R}^2}, \underbrace{|xy|}_{\mathbb{R}}\Big) \in \mathbb{R}^3$$

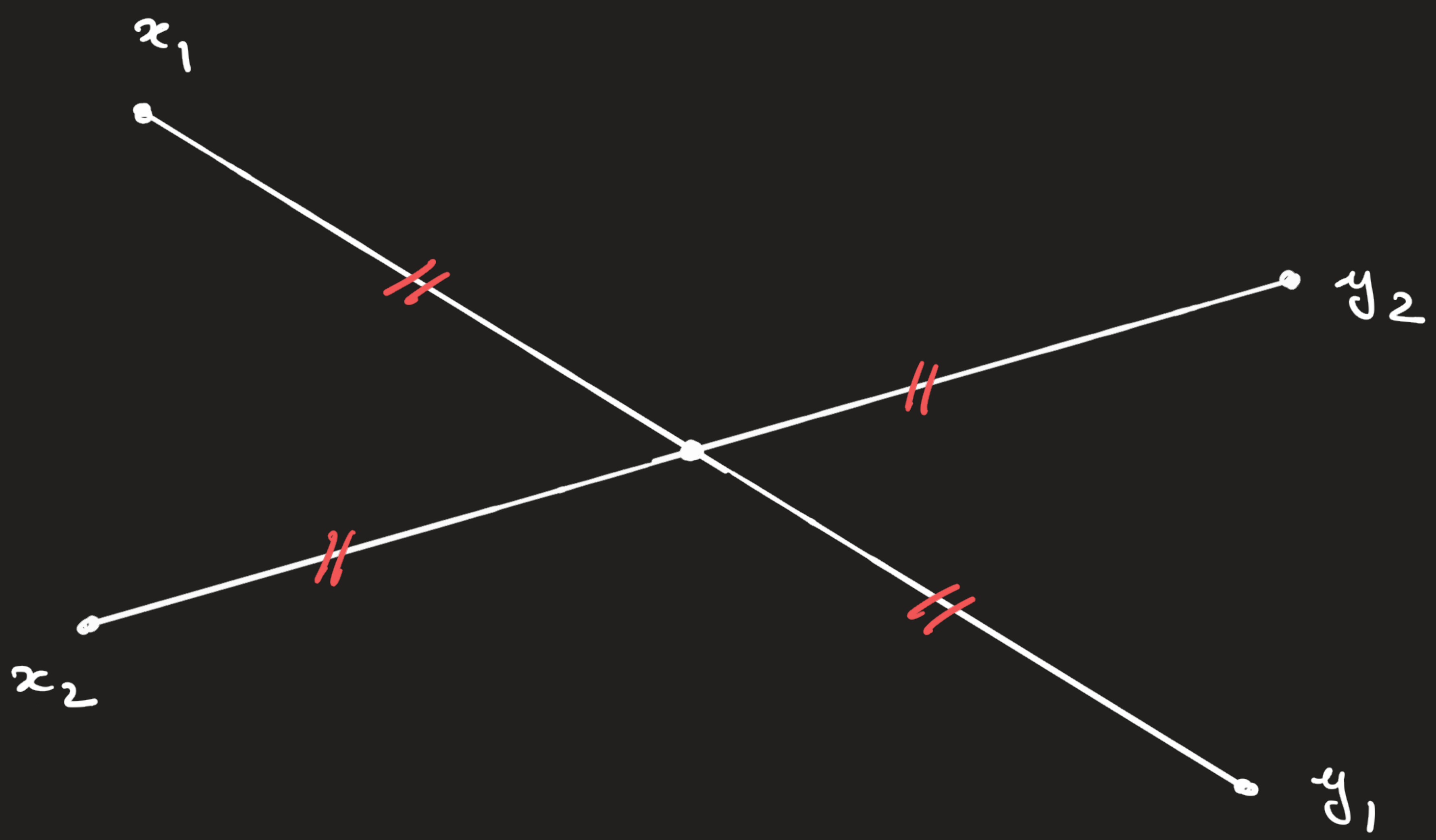
$$\phi(x,y) = \left(\frac{x+y}{2}, |xy|\right)$$

Si Φ n'est pas injective

$$\Phi(x_1, y_1) = \Phi(x_2, y_2)$$

$$\begin{cases} \dfrac{x_1+y_1}{2} = \dfrac{x_2+y_2}{2} \\ |x_1y_1| = |x_2y_2| \end{cases}$$

□

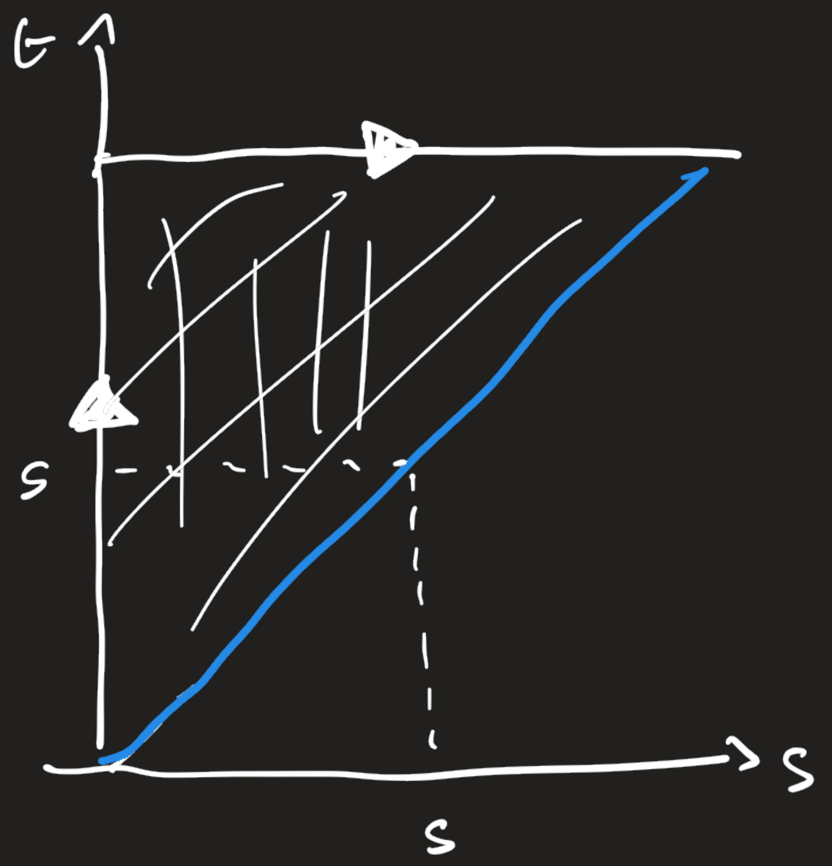


$$\frac{x_1 + y_1}{2} = \frac{x_2 + y_2}{2}$$

$$|x_1 y_1| = |x_2 y_2|$$

Si Φ est injective alors on peut construire un plongement de $\mathbb{R}P^2$ dans $\mathbb{R}^3$.

Théorème (Jordan-Brouwer) Une surface compacte qui peut être plongée dans $\mathbb{R}^3$ est nécessairement orientable.



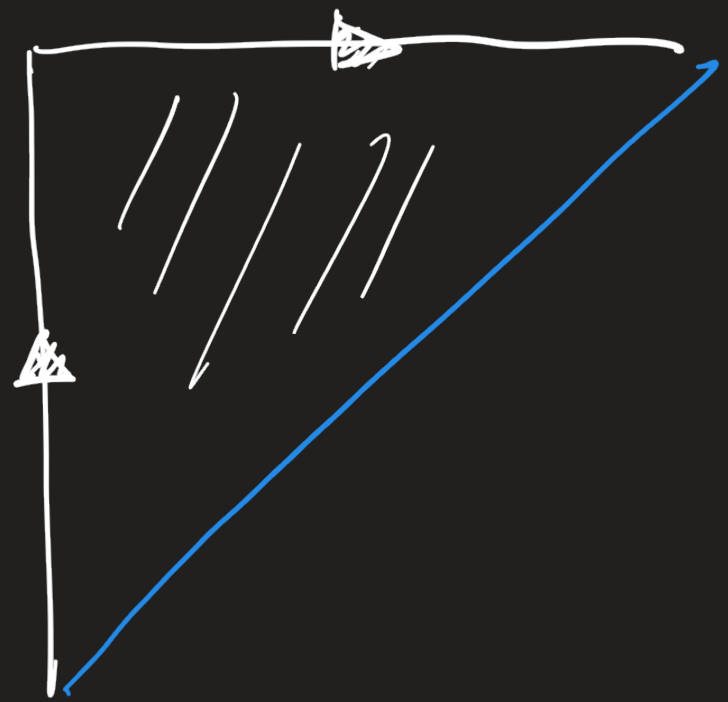
(x, y)

$(\gamma(s), \gamma(t))$

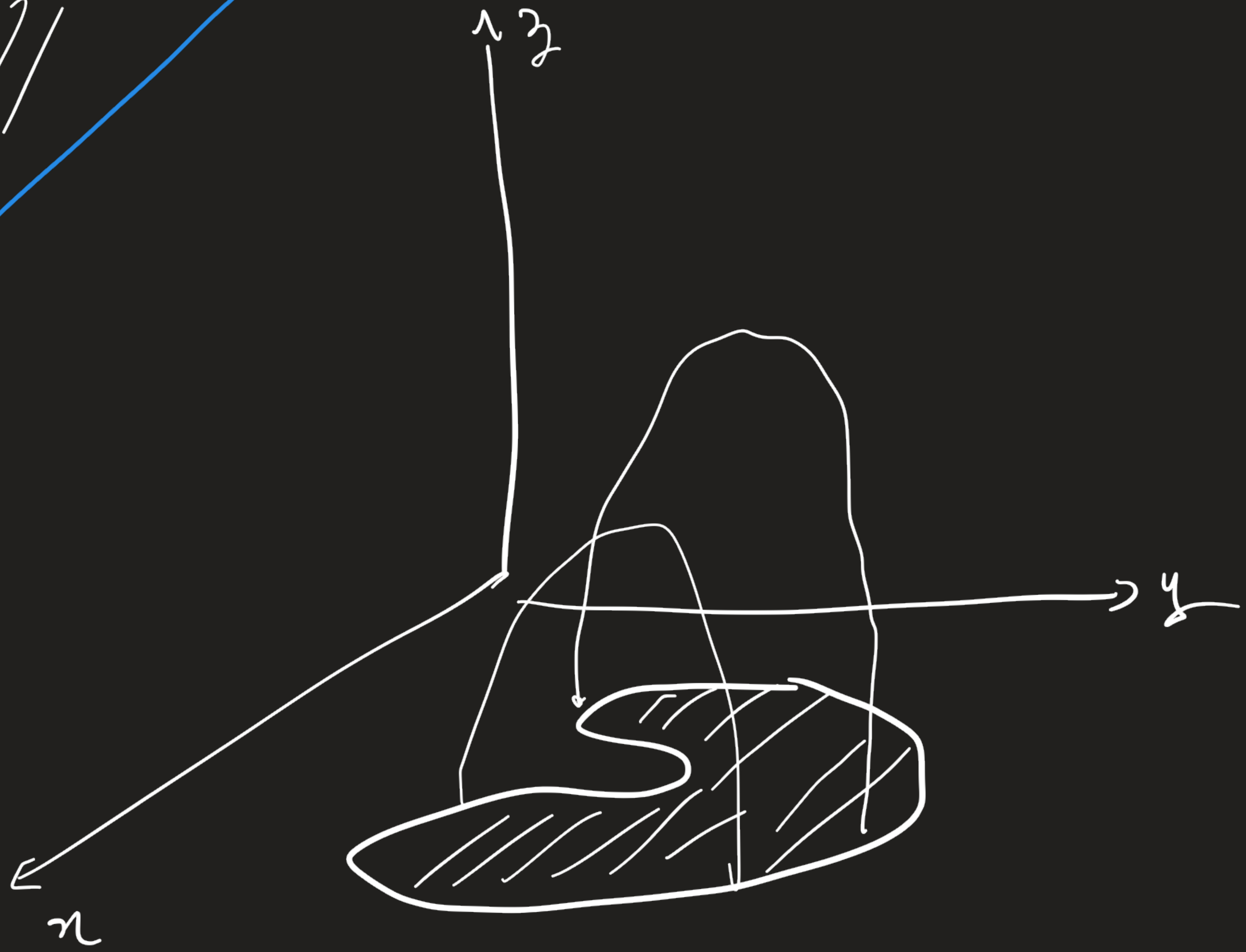
$\gamma(s), \gamma(s)$

$$\phi(\gamma(s), \gamma(t)) = \left(\frac{\gamma(s)+\gamma(t)}{2}, |\gamma(s)\gamma(t)|\right)$$

$$\phi(\gamma(s), \gamma(s)) = (\gamma(s), 0)$$



$$\phi(x,y) = \left(\frac{x+y}{2}, |xy|\right)$$




That's all Folks!